6.013 Electromagnetics and Applications
Spring 2009

ELECTROMAGNETICS AND APPLICATIONS

Electromagnetics and Applications

- Maxwell's equations: statics, quasistatics, and wave phenomena
- Applications: wireless, media, circuits, forces and generators, computer speed, microwaves, antennas, photonics, acoustics, etc.

Mathematical Methods

- Partial differential and difference equations, phasors, vector calculus

Problem Solving Techniques

- Perturbation, boundary-value, and energy methods; duality

Academic Review

- Mechanics, quantum phenomena, devices, circuits, signals, linear systems

Capstone Subject—Professional Preparation

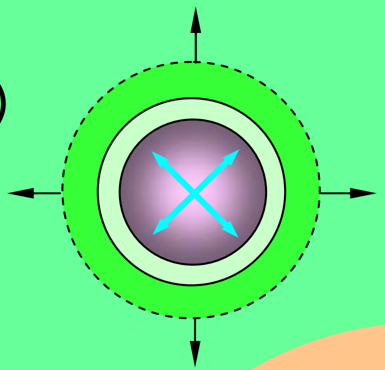
Follow-on Subjects:

Electromagnetic waves: 6.632, Quasistatics: 6.641

ACOUSTIC ANTENNAS

Monopole Radiators:

$$\nabla p = -\rho_o \frac{\partial \bar{u}}{\partial t} \quad [\mathrm{Nm}^{-3}]\ [\mathrm{kg\ m}^{-2}\mathrm{s}^{-2}]$$



Wave equation: $(\nabla^2 + k^2)\underline{p} = 0^*$ $\quad (k = \omega/c_s)$

$\partial/\partial\theta = \partial/\partial\phi = 0$ $\quad$ (radial source)

Yields: $d^2\underline{p}/dr^2 + (2/r)d\underline{p}/dr + k^2\underline{p} = 0$

Equivalent to: $d^2(r\underline{p})/dr^2 + k^2(r\underline{p}) = 0$

General solution: $r\underline{p} \propto e^{\pm jkr}$

Radiation outward: $\underline{p}(r) = (A/r)e^{-jkr}$

Velocity field u: $\underline{\bar{u}}(r) = -\frac{\nabla \underline{p}}{j\omega\rho_o} = \hat{r}\frac{A}{\eta_s r}\left(1 + \frac{1}{jkr}\right)e^{-jkr}$

Far-Field: kr >> 1 $\Rightarrow$ r >> $\lambda/2\pi$):

$p(r) = (A/r)e^{-jkr}$ $\qquad \bar{u}(r) = (A/r\eta_s)e^{-jkr} = p(r)/\eta_s$

Near-Field Radiation:

Since $\nabla \bar{\underline{p}} = -j\omega\underline{\bar{u}}$ therefore:

$p(r) = (A/r)e^{-jkr}$ $\qquad \bar{u}(r) = (-jA/r^2k\eta_s)e^{-jkr} = -jp(r)/\rho_o\omega r$

"Velocity mikes" close to the lips boost lows; need $\times\omega$ compensation

$$* \left(\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right)$$

ACOUSTIC ANTENNAS (2)

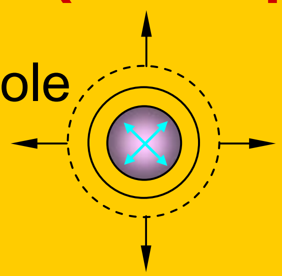
Antenna Gain G(θ,φ), Effective Area A(θ,φ) [m²]:

$$G(\theta,\phi) = \frac{P_r(\theta,\phi)}{(P_t / 4\pi r^2)}$$

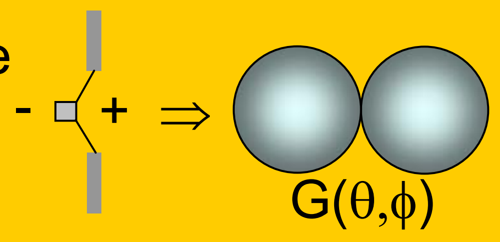
$$P_{received} = I(\theta,\phi)\, A(\theta,\phi) \text{ [W]}$$

Antenna (Loudspeaker, Microphone) Configurations:

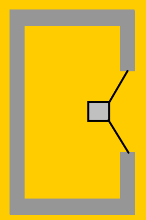
Monopole

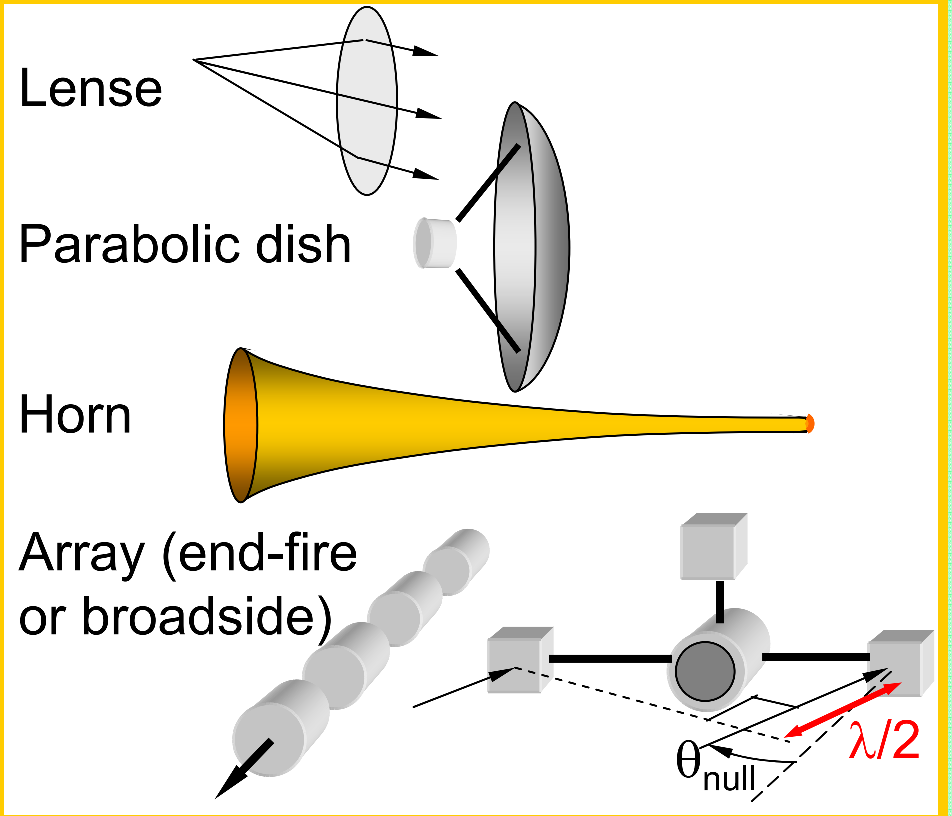


Dipole



Baffled monopole





ACOUSTIC RESONATORS

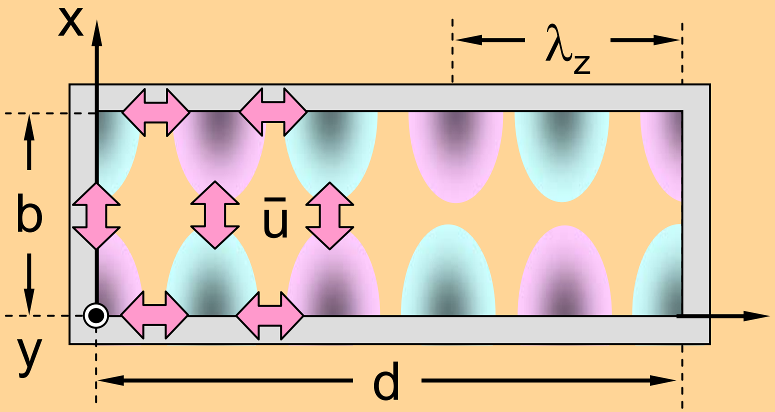
A_{mnp} Resonances of a Box:

$$m\left(\frac{\lambda_y}{2}\right) = a \Rightarrow \lambda_y = \frac{2a}{m}$$

$$n\left(\frac{\lambda_x}{2}\right) = b, \quad p\left(\frac{\lambda_z}{2}\right) = d$$

$$\left(\nabla^2 + k^2\right)\underline{p} = 0$$

e.g., $\underline{p} = p_o e^{-jk_x x - jk_y y - jk_z z}$



$$k^2 = \frac{\omega^2}{c_s^2} = k_x^2 + k_y^2 + k_z^2 = \left(\frac{2\pi}{\lambda_x}\right)^2 + \left(\frac{2\pi}{\lambda_y}\right)^2 + \left(\frac{2\pi}{\lambda_z}\right)^2$$

Resonant Frequencies of the A_{mnp} Mode in a Box:

$$f_{mnp}^2 = \left(\frac{\omega}{2\pi}\right)^2 = c_s^2\left(\frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2} + \frac{1}{\lambda_z^2}\right), \text{ where } \lambda_x = \frac{2b}{n}$$

$$f_{mnp} = c_s\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2 + \left(\frac{p}{2d}\right)^2} \quad [\text{Hz}]$$

f_{000} = 0 Hz (constant pressure)

f_{001} = 340/2d $\Rightarrow$ 170 Hz for a one-meter closed pipe

RESONATOR MODAL DENSITY

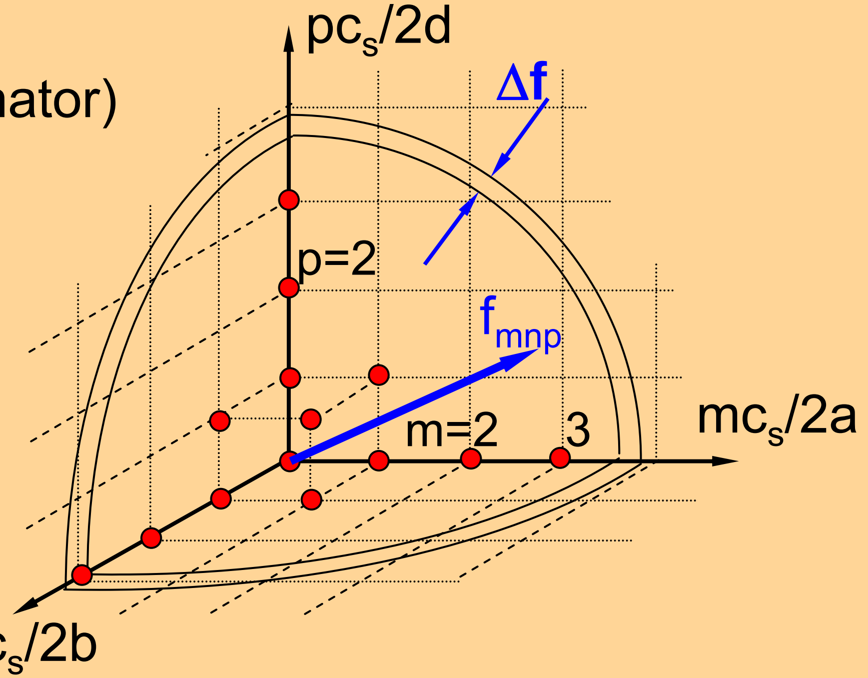
Modal Density in Rectangular Resonators:

Recall: $f_{mnp} = c_s\sqrt{(\frac{m}{2a})^2 + (\frac{n}{2b})^2 + (\frac{p}{2d})^2}$ [Hz]

Each cube has volume = $c_s^{\,3}/8V$
where V = abd (volume of resonator)

Number of modes in $\Delta f \cong$
(Volume of shell)/(vol. of cell) $\cong$
$4\pi f^2\ \Delta f/[8(c_s^{\,3}/8V)] \cong$

$4\pi f^2\ \Delta f\ V/c_s^{\,3}$ modes in Δf



Example:

Bathroom 3×3×3 meters
$\Rightarrow$ lowest $f_{100} = c_s/2a \cong 340/6 \cong 57$ Hz
Modal density at 500 Hz $\cong 4\pi \times 500^2 \times 1 \times 3^3/340^3 \cong 2$ modes/Hz
How can we select just one mode when we sing a single note?

EXCITATION OF RESONATORS

TEM Resonators (with loss):

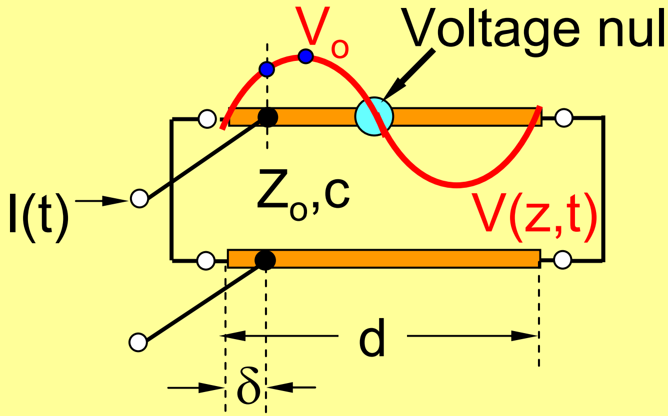
$I(t) = I_o \cos \omega_o t$

$V(\delta,t) = V_o \cos(\omega_o t + \phi) \sin(2\pi\delta/d)$

$\phi = 0$ exactly at resonance

$P_{in}(t) \cong I_o V_o \cos^2(\omega_o t) \sin(2\pi\delta/d)$ [W]

$= 0$ at voltage nulls



Cannot excite TEM_m modes by driving current into voltage nulls! (Or by voltage sources in series at current nulls). $P_{in}(t) = 0$ in both cases.

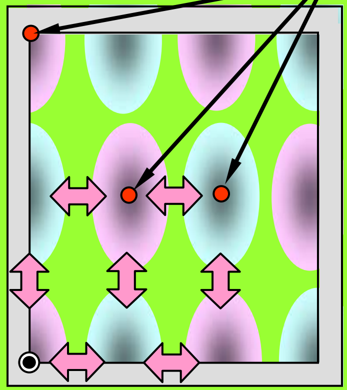
Acoustic Resonators:

$I[Wm^{-2}] = p\bar{u} \bullet \hat{n}$

Cannot excite acoustic modes with:

velocity sources at pressure nulls ($p_k = 0$), or

pressure sources at velocity nulls ($v_k = 0$)



Bathroom Opera:

Mouth ≈ velocity source

Place mouth near a pressure maximum of desired mode

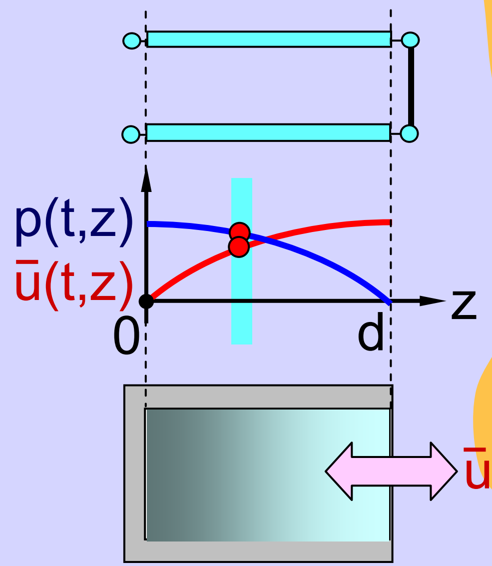
HUMAN ACOUSTIC RESONATORS

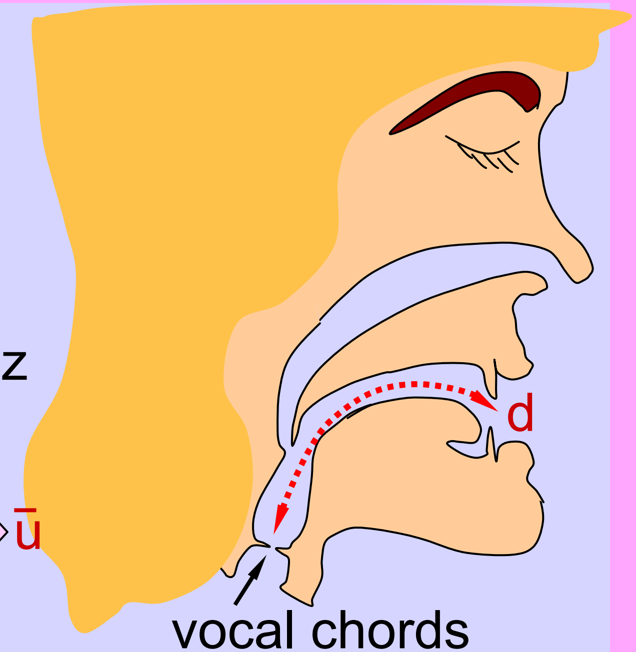
Human Vocal Tract:

$f_1 = c_s/\lambda_1 = c_s/4d$

$= 340/(4 \times 0.16)$

$= 531$ Hz





Higher Resonances:

$f_2 = 3f_1 = 1594$ Hz

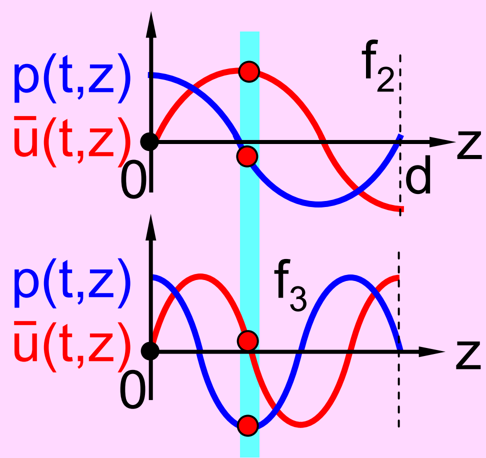
$f_3 = 5f_1 = 2655$ Hz

Energy Densities at Location “ ”

At f_1: $w_p \cong w_u$

At f_2: $w_u >> w_p$

At f_3: $w_p << w_u$



RESONANCE SHIFTS IN HUMAN VOICES

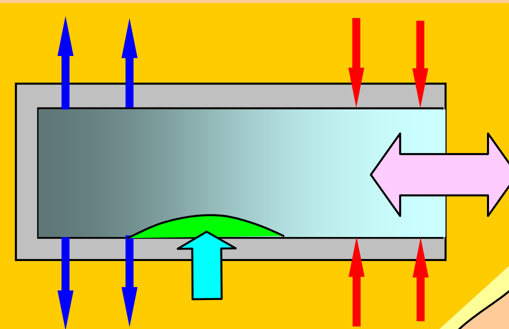
Human Vocal Tract:

Average force exerted by waves:

Outward at maximum $|\underline{p}|$ (max w_p)

Inward at maximum $|\underline{u}|$ (max w_u)

(Bernoulli force)

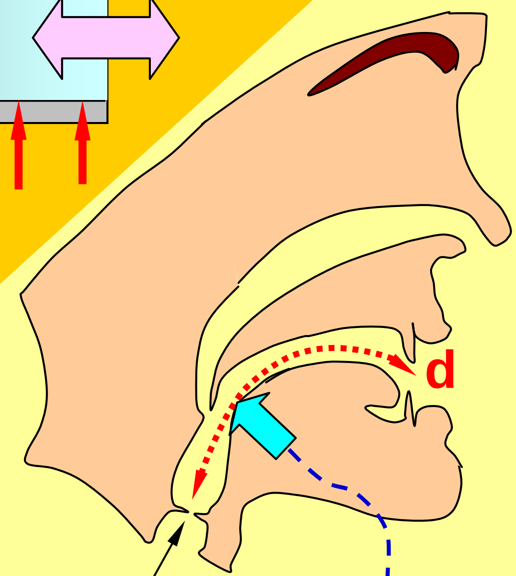


Resonator Total Energy $w_T = nhf_o$:

Pressing inward at p_{max} increases w_T and f_o

(Phonon number n = constant for slow changes)

Recall: pressure {N m^{-2}] $\propto$ energy density [J m^{-3}]



Resonance Perturbations:

$$\frac{\Delta f}{f} = \frac{\Delta(w_p - w_u)}{w_T}$$

$$f_o \propto c_s \propto \sqrt{\frac{\gamma P_o}{\rho_o}}$$

Tongue position determines vowel

$w_p >> w_u$ at f_3

$w_u >> w_p$ at f_2

$w_p \cong w_u$ at f_1