STUDY SKILLS IN MATHEMATICS

The following document is intended to help you adapt to what may be different styles of teaching and learning as compared with your experiences at sixth-form.

It is not intended to be read from beginning to end in a single sitting. Instead, dip into it and read about the topic which currently interests or concern you – whether it is making best use of lectures, trying to tackle problems, revise for exams, or whatever.

To help you find your way through it, the document contains a brief summary of each of the sections listed in the Contents. Then, at the beginning of each section, there is a page of "key points" to help you get an overall view of the detailed advice and suggestions which follow.

Acknowledgement

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HEALTH WARNING

You can have too much of a good thing. If all the advice in this pack seems rather daunting do not despair. You will soon get the hang of things at university and succeed like thousands of students before you. No matter what happens, **don't panic!**

Also, this pack gives **general guidance** only. You may well find you are required to do things in a slightly different way for different modules. Whenever this is the case the module tutors should tell you, but **if in doubt**, ask them!

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ABSTRACTS

Introduction
Analyse how you work most effectively in mathematics and try out new ways of studying; discard those you can't adapt to suit you. Higher Education can be very different from anything you have met before: develop your study methods to cope with this new situation.

Be organised
Make life easier for yourself by planning your time and packaging your work so that it can be done in the time: be flexible but watch those deadlines.

Getting to grips with Mathematics
If you get stuck trying to solve a problem or learning a theorem, don't panic or wait for inspiration (well not for too long anyway!): try the suggestions in this section.

Learning from lectures
Be active in lectures and help yourself to concentrate by constantly questioning and interpreting what's said. Afterwards don't leave your notes to gather dust, work them up and do any follow-up problems, exercises, reading while the lecture is still fresh in your mind.

Other ways of learning
Get the best out of tutorials, textbooks (not as easy as you might think), calculators, computing facilities and libraries. Use tutors and co-operate with follow students.

Revision techniques
'Suss out' what is needed to pass your exams and orientate your revision with this in mind. Decide on a revision timetable (incorporating plenty of flexibility of course) and experiment with different ways of revising (remember mathematics is best learnt by doing). What about getting together with fellow students?

Winning at examinations
Exam nerves are normal: there's no escape from them for most of us. Win through with careful preparation and a systematic approach to the paper. Stay cool and don't panic.

Winning at coursework and projects
Don't leave it too late to start. Talk to others and give yourself plenty of time to finish and polish your final offering.

INTRODUCTION

The aim of this booklet is *to help you acquire the skills and techniques necessary for effective study in mathematics*. It contains a lot of simple, straightforward advice which many students have found helpful in the past, but you will find that not everything here suits you. The suggestions are offered only as guidelines, not as imperatives: some of them will be inapplicable to particular branches of mathematics while others will be inappropriate for certain types of students.

It is important to *find out how and where you work best* and so time invested during your first term in experimenting with study methods will pay off later. You will need to persevere with new approaches to learning, for example for taking detailed notes during lectures and annotating them suitably, and in solving set problems on your own. You can't expect immediate perfection, so be patient if you are slow at first.
The transition from school to higher education can be traumatic. Not only are there inevitable and substantial changes in your social and personal circumstances – feelings of strangeness, loneliness and even euphoria are common – but *you will also find dramatic changes in the teaching/learning regime*. At school your work was probably directed by a teacher who was in close personal contact with you and able to give you continuous encouragement and feedback – good and bad – on your progress. In Higher Education, although the pace of work is much greater, it is left very much to you to decide your own level and direction of study. You will find much less direct personal contact with staff and less direct feedback on your academic performance; it is often difficult to know 'how you are standing'. *You will have to find ways of motivating yourself*, of keeping yourself going, *and checking on your progress*. Try to find ways of rewarding yourself when you have made some progress – even if it is just by having a cup of coffee. Avoid those situations that distract you from work or commitments that will be too time consuming.

Learn what sort of person you are and how you can maximise your chances. Many institutions offer *projects* and *continuously assessed courses* as well as the traditional lecture course followed by a written examination. You may have to make sensible choices of what is suitable for you. For example, if you tend to panic in examinations, it might be a good idea to consider including a project or continuously assessed course in your options. If you are unsure about what to choose and need some help do not hesitate to go for advice to your tutor or some other appropriate member of staff.

BE ORGANISED

A little time and thought spent trying to improve your study methods will pay handsome dividends—remember that mathematics often needs a different approach to other subjects.

A STRATEGY FOR IMPROVEMENT

Think about how you study:

- *when do you work best?*
  (surely not after a good lunch or at 3am?)

- *how long can you stick at it*?
  (do you find a number of short breaks better than one or two long ones?)

- *how can you reward yourself?*
  (a glass of beer after working is generally a better strategy than one to 'get you going')

- *have you got a 'special place' for working?*
  (very few of us can study well in a horizontal position)

- *do you make best use of the day?*
  (work hard and play hard—there's time for both)

- *when are your deadlines?*
  (they do sneak up)

Try out the suggestions in this booklet:

- *some will be 'old hat' to you*
  (well we don't claim to be original!)

- *some will be useful*
  (adapt these to suit yourself)

- *some won't work for you*
  (not surprisingly, study methods are very personal)

MOST IMPORTANT OF ALL

- *find out what works for you and develop your own individual approach to learning and doing mathematics*
  (if there are options available to you within your course try to choose those in which you are likely to do best)

BE ORGANISED

It is important that *regular habits be established quickly and adhered to as far as possible*. Within this pattern it should be recognised that various modules may occupy different amounts of time but these difference should not get out of proportion. Be wary of spending a disproportionate amount of time on work such as computing, projects and modules which are continuously assessed, important though these are.

Experiment and find out what times of day suit you best for studying, given the constraints of your timetable and set meal times. Change to a different type of work if you get bored.

You may be able to keep going for much longer when solving problems. The odd hours that occur during the day between lectures and tutorials are particularly valuable because they offer the opportunity of working in the environment where staff, other students, micro-computing lab and library facilities are all available.

A common problem is feeling overwhelmed by the vast amount of work that has to be done leading to a reluctance to get started. *It may be worth 'packaging' your work into manageable amounts*. Try spending a few minutes planning your proposed work before actually launching forth. A couple of minutes deciding on how much time you are going to spend studying (you might decide on no more that 30 – 60 minutes without a break in order to get started) and precisely what you are going to work at can make the prospect of starting much more palatable. It also helps to have a 'special place' for work that is physically comfortable yet free from distraction. As you work, try to get into the habit of asking yourself questions as this is a good way of keeping your interest alive and identifying areas where you are weak. But do try to finish on a successful note then maybe next time it will not be quite as hard to get started!

Although many people find the pressure of a deadline for the completion of an assignment helpful to motivation and concentration, it is essential to allow yourself enough time so that the work you submit does you justice. For computing assignments the earlier you start the better; it takes longer than you think to understand how to use a package or to write a working program.

A sample timetable together with suggested use of spare slots is given in the Appendix to show you how to organise your time effectively.

Keep in regular touch not only with your tutors but also with fellow students—*you will learn a lot from helping each other*; for example, by following through a proof or doing an exercise together.

Think carefully before you choose any optional modules. Make sure that your choice includes those options which will be of help to you in your subsequent career and those which are necessary prerequisites for any subsequent modules you may wish to take.

Try to choose those options which appeal to you and in which you feel you are likely to do well. However, if you want to choose an option outside mainstream mathematics—say French or Accountancy—think carefully. For example, in some institutions the examinations marks in an Arts subject tend to lie in the middle range; if you are below average, this may act to your advantage, and conversely. If the option is open to other students with a more relevant background than yours, this may make things difficult for you. Options involving project work and reports may sometimes be more suitable for you and for your future career. If you feel you do not do yourself justice in examinations, this may be a very wise choice.

A PLACE TO STUDY

- It may be difficult to get some peace in hall. Try a ‘Do Not Disturb’ sign on the door.
- A firm chair and a desk can help concentration.
- You need good lighting directed at the page (e.g. an anglepoise lamp).
- It needs to be warm but not hot.
- If you feel sleepy try opening a window.

What are your ideal conditions for working? It could be loud music or very quiet. It could vary depending on what you are doing. When you find a good place or way of working it is useful to stick to it and associate it with work.

GETTING TO GRIPS WITH MATHEMATICS

Get started by:

- *identifying key words/concepts*
  (check with your lecture notes)
- *making sure you know the mathematics involved*
  (you need to know the fundamentals)
- drawing a diagram
  (often helps to 'see' the problem)

Notations:

- *get to know the meaning of technical words and your lecturer's own abbreviations*
- *get to know the Greek alphabet and special symbols*
  (e.g. ξ, ζ, ∂, Υ, $\aleph_0$)
- *make a list of the meanings of technical words*
  (e.g. proof, axiom, necessary, sufficient…)

Got stuck? Have you:

- *re-read the question?*
  (check you understand what is wanted and look for hidden clues)
- *looked for similar problems?*
  (your lecture notes and textbooks are obvious sources)
- *tried to simplify the problem?*
  (putting in numbers often helps)
- *written down your thoughts?*
  (brainstorming might help you to spot connections and omissions)
- *tried to explain why you can't get started to a fellow student?*
  (once you try to explain your difficulty clearly to another person the way forward sometimes becomes obvious)
- *asked your tutor?*
  (quite likely s/he will be reluctant to give you the complete answer but may give you a useful tip to get you going)

Still stuck? Then:

- *take a break*
  (maybe the way forward is staring at you and you only need to stand back from it!)

GETTING TO GRIPS WITH MATHEMATICS

One learns, and loves, mathematics by doing it, not just by reading it. Therefore tackling exercises and problems is an essential part of learning mathematics and by far the most efficient way of consolidating and checking your understanding of the course. Books and notes should always be studied with pencil and paper to hand.

Notations and conventions

Get to know the meanings of any special symbols (such as ξ, ζ, ∂, Υ, $\aleph_0$) including the Greek alphabet and how to say them. Make your own list at the front of your notes. Try to find out the different meanings of words such as 'theorem', 'axiom', 'definition', 'proof', 'example', 'whenever', 'unless', 'if', 'only if', 'necessary', 'sufficient', 'for some', 'for any', 'for each'. The distinction between a general proof and a particular example is commonly misunderstood. Be aware that many words like 'bound', 'limit' and 'infinity' have technical meanings superficially similar to but actually quite different from common usage.

Exercises

Exercises are questions intended to train you to become skilled in the straightforward application of techniques and/or to give you examples of concepts which have been introduced in lectures. If possible, *you should try them before going to any class based on them*, so that you can raise your difficulties. It is hardly sensible to spend your time in class working through questions which you can do, only to find out later that you have difficulties with some of the others. You should try to complete all the exercises set, not just those which have to be handed in.

Note that unless a specific numerical value is asked for it is normally acceptable to leave mathematical constants and functions, such as π, e, or $\cos$, in your answers. If you are asked to give your answers to a prescribed degree of accuracy do so (and not to a greater degree as given by your calculator).

Problems

Problems are not always straightforward: they usually require some insight and creative use of the mathematics concerned. You may find that you are unable to do them all and that some of them you cannot even start! *There is no recipe for solving problems but here are some of the things you can try if you get stuck*. The following sequence indicates how you might go about tackling the problem sheets.

To begin with:

1. Have lecture notes, relevant textbooks and calculator at hand. Learn what your pocket calculators can do so that you can use it to maximum advantage but do not become over-dependent on it.

2. Read the entire question carefully and identify the key concepts involved, (e.g. by comparison with the key word list from your notes).

3. Identify the mathematics likely to be involved and make sure that you know and understand it. Re-read the relevant section of your notes if necessary. Look for similar problems.

4. Draw a picture if the problem allows it. In applied mathematics diagrams are almost always essential, (e.g. to show the forces acting on a body), but they can also help in other areas of your work. Experiment with pictures, diagrams, patterns—they often provide a real breakthrough. Make use of any software packages which may be available to help you experiment more freely with different diagrams.

5. If the problem is a general one, look at special cases or formulate and solve a simpler problem first.

6. If the problem is abstract, look at concrete examples, (e.g. put numbers in place of letters), Take specific examples of sequences or sets and so on. If possible draw a picture or diagram. Calculators and/or software packages will again be of use here.

7. Think around the problem and write down your thoughts, (why not try producing a 'patterned note' of the problem?). What you write down often gives you other ideas and helps to build up a solution step-by-step. What you don't write down often vanishes into thin air and is not remembered at the vital moment.

8. Don't be afraid to experiment and perhaps go up blind alleys. Although the final solution should be logically ordered you may find it helpful in developing it to work backwards from the result to be obtained (if you are given it) as well as forward from the data of the problem. Be prepared to gloss-over details to start with in an attempt to obtain the broad outline of a solution. You may be able to fill in the details later.

If you have difficulty starting the problem:

9. Re-read the question and make sure you know what all the words mean and that you can give examples of their use.

10. Notice that key words like 'deduce', 'hence' and 'otherwise' often suggest ways of doing problems.

11. Make a list of key concepts involved in the statement and re-read the relevant theory.

12. See if you can do easier problems of the same type.

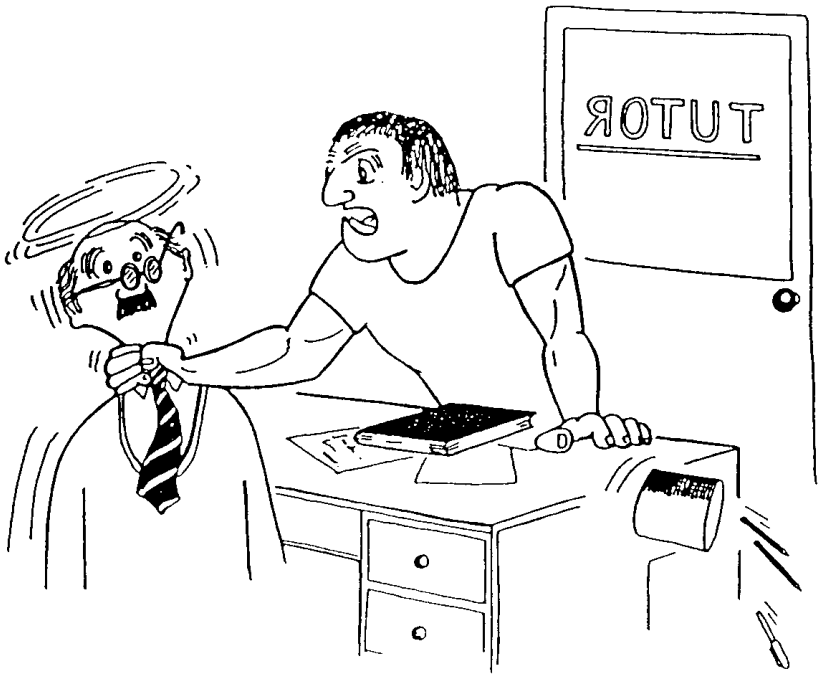
13. If you are still in difficulty then look for a fresh approach (e.g. from textbooks containing similar examples).

14. Discussion with fellow students often produces new insight. Explaining ideas to others will help to clarify your own thinking. For both exercises and problems, it is often helpful to work with others. Don't be afraid to share ideas; you will find that others share your difficulties but each can contribute new ideas or understandings. At the same time, try to develop your own thinking: there is little value in simply copying someone else's work. Any work to be handed in for assessment should be your own, of course.

15. If you feel you have reached a dead-end, then don't give up—in the course of solving a problem new key concepts may appear and the suggestions above may need to be retraced.

16. If you get really stuck with a problem, don't sit at it for too long. Try something else, have a break, or sleep on it (but not in exams!). *It is important to know when to stop trying: and note your difficulties for discussion at the next problem class or tutorial*.

17. Do not be afraid to ask your tutor or lecturer for help.



Do not be afraid to ask your tutor for help

Once the problem is complete:

18. Try to place the problem in context relative to the whole module; look back at what you have done, check the arguments and look for links between the result and the method with the rest of the module

19. In writing out the final solution, state what the symbols represent and pay careful attention to the logical flow of the argument. Read it back to yourself to see that it makes sense. Check your working, with numbers if possible. Keep the question and your solution filed for future reference at revision time.

20. Handing in work is most important, it's the only way you'll find out whether your ideas are right, whether you've understood the problem and whether your solutions are correct (even if the final answers look right). Ask for, and pay close attention to, comments on your work.

21. If model solutions are provided then make use of them. Compare your solutions with those given. Sometimes you may learn more from a model solution to a problem for which you have found a correct solution than you will from solutions to problems which have baffled you.

22. If in the course of attempting problems which are set you feel that there are shortcomings in either your lecture notes or your understanding, then make a note of these and bring them up at the next tutorial or problem class. You may also find it helpful to attempt further problems and exercises from relevant textbooks.

23. If you experience considerable difficulties and many other students also do, then approach your lecturers and tutors collectively. If the problems persist then approach your student-staff committee.

Get ready for the next problem:

24. There is an Open University Foundation course book (see booklist) with many useful ideas on the techniques of problem solving—use it. Polya (1990) has written a book entitled 'How to solve it' which you might also find helpful.

25. Learn the art of drawing a good, big picture, especially in applied mathematics. For example, with a problem about a ball moving under the influence of forces such as gravity and air-resistance, draw a picture including all forces, velocities, and accelerations (vertical and horizontal), and state the co-ordinate framework being used.

26. Make good use of any software package that is provided to go with the course. If none is mentioned by the lecturer ask him or her whether any is available.

Theorems

Understanding theorems is often a source of difficulty. When tackling a new theorem, *find out first of all what it says and how it is used in the subject*. Next *try to identify the three or four main steps in the proof* to get a broad outline of how it works. It may be helpful to put these on the 'key facts' cards or in the margin. Finally learn the details.

Two of the more difficult things in learning mathematics are the construction of proofs and the appreciation of theorems. These are inseparable from thinking mathematically. Practice and experience have no substitutes but you will find the following useful:

Use diagrams and pictures to explore the truth or falsity of statements through explicit illustrations.

1. Study lecture notes and set textbooks in order consciously to identify the assumptions and the conclusions of theorems.

2. Try to locate when each assumption is used in a proof. It may help to highlight such places in a particular colour. You can sometimes appreciate why a particular assumption is needed by investigating a counter-example to the theorem in which the assumption does not hold.

3. Attempt proofs of 'parallel' theorems mentioned in lectures or left as exercises in the lecture notes or in the set textbook. Look for simple special cases. Familiarity with the simple makes the complex easier.

4. Gain practice in the various methods of proof. Identify the ideas in proofs and note their re-appearance.

LEARNING FROM LECTURES

Lecture notes provide the most permanent record of a module—so do experiment and develop your own note taking technique.

NOTE TAKING

Exploit the obvious:

- *arrive in good time and sit where you can see well*
  (with pens, pencils, ruler, etc. and your current notes for the module)

- *prepare yourself mentally for the lecture; easier if you have a module outline*
  (ask yourself questions about what's going to come up)

- *number the pages and add the date*
  (avoid the frustration of getting your notes in a muddle)

- *leave wide margins (or a blank facing page) for later annotation*
  (working on your notes afterwards is essential)

- *use headings to distinguish between major and minor results, theorems, etc.*
  (make sure you can see the wood from the trees)

- *use a different colour to highlight key points*
  (identify the key points and get an overall picture of what's going on)

- *ask the lecturer to repeat or slow down if necessary*
  (you must get down the presented material)

Accuracy is important

- *you can waste a lot of time by trying to justify an error in a line of mathematics or an incorrect formula; be brave and* ***ASK IMMEDIATELY*** *if you suspect a mistake on the board.*

Make a conscious choice of what you write down:

- *everything on the board*
  (this is the absolute minimum)

- *the more important things said*
  (judging what's important makes you concentrate and think)

- *but not everything the lecturer says*
  (it's impossible although some students try!)

LEARNING FROM LECTURES

Note taking and note using

At the beginning of a module it is a common misconception that students should understand everything that is presented during each lecture at the time when it is given—this is very rarely possible in mathematics. Nevertheless, it is vital to make a thorough record of each lecture as a basis for reference and further study when you check through the details of the manipulations, absorb the overall strategy, learn the definitions, etc.

The most permanent information about any lecture module will be the set of notes which you have obtained by attending it. In addition, you will have to make notes yourself from reading relevant textbooks. *It is essential that these notes should be as clear as possible and arranged so that you can find information in them with ease*.

Acquiring the skill of making good lecture notes is essentially a matter of commonsense and practice. Some of the suggestions given below may seem so obvious that they hardly needed to be stated but it is amazing how easily they can be ignored or neglected. You should always:

1. Arrive at the lecture in good time and with all of the necessary equipment. As well as normal pens, pencil and paper, *it is a good idea to have a least one pen of a different colour in order to highlight important points and theorems*. Sit as close to the front as possible. You will be able to see better and be less liable to distraction.

2. Take your current notes to lectures and tutorials and remember to file different modules separately.

3. Number the pages of your notes and make the place where each lecture starts with the date.

4. *Leave plenty of space so that you can annotate your notes* easily when you are studying them afterwards—either leave margins or use only one side of the paper so that the page facing each page of notes is blank.

5. Use headings carefully to distinguish major from minor results, proofs from examples, theory from techniques.

6. *Make sure that your notes are accurate*. You can waste a lot of time subsequently if, for example, you have written down an inequality the wrong way round or written x instead of x^2 say. Whereas a mistake in an English sentence may be quite easy to rectify later by the sense, it is often much harder to spot and correct a mathematical error. So be particularly careful to copy formulae and such like exactly.

7. Your lecturer may use abbreviations not found in textbooks: make sure you understand the meaning of these.

In the lecture

A constant problem in lectures is that you are required to listen to the lecturer, look at illustrations, read explanations and make your own notes, all at the same time. To do this successfully requires concentration and skill. *You must make sure that you write down at least everything that the lecturer writes on the board or displays on the overhead projector (OHP)*. Some of this material will remain visible for a little while after it is written—long enough to allow you to copy it down; alternatively the lecturer may provide duplicated copies of the material. However, what the lecturer is saying is heard only once; you do not have much time to decide what part of it to write down. A balance

must be achieved between taking no notes of the spoken word and trying to make a word-for-word transcript, and the art of doing this must be obtained as quickly as possible.

When you don't understand the lecturer, don't panic: keep taking notes and seek help as soon as possible from the lecturer, your tutor and other students. *Don't be afraid to ask questions* during or at the end of the lecture; usually other students don't understand either and will admire your courage. Lecturers, like other human beings, often make mistakes—sometimes 'slips of the pen' and sometimes mathematical errors—so do point these out as soon as you spot them! And remember that difficult ideas become easier to understand if you work at them through reading and doing problems.

If a lecturer uses a lot of graphical illustrations, don't be afraid to ask for printed examples of what you have seen on the screen, in the case of computer demonstrations, ask how you may reproduce the material in the computer laboratory.

You should identify the style of lecturing. Here are some common types:

1. **Continuous written presentation without discussion**

   This is a very formal style of lecturing which is still encountered. You must copy everything down and ensure your record is accurate. Don't hesitate to ask the lecturer to slow down, to write more legibly or to re-display slides if you are having difficulty in keeping up. It is essential to re-read the material as soon as possible afterwards to consolidate it, since there will be no chance during the lecture.

2. **Written presentation punctuated by spoken discussion**

   It is essential to be able to distinguish between the formal notes and the discussion—ask for clarification if necessary. Ensure accuracy etc. as for case 1 above. Précis the discussion by making comments at appropriate places in the margin.

3. **Two stage lecture** (Class discussion followed by formal lecture or vice-versa)

   Don't be afraid to participate in the discussion even if you may feel that you will make a fool of yourself. It is certainly not necessary to take notes during the discussion stage—listen and participate. The comments in case 1 apply to the formal lecture.

4. **Lecture supported by set books and/or duplicated notes**

   Read the relevant section before going to the lecture. Make notes of any points causing trouble so that you can raise these difficulties during the lecture. Supplement notes with written comment from the lecturer. Above all, take your notes or books to the lecture for reference.

5. **Vague Style**

   Don't be afraid to ask questions such as:

   "Could you write that on the board?"

   "Could you explain that again, please?"

   Try to get at least a list of the main topics covered in the lecture. If all else fails then consult (or ask for) a recommended text.

Unless you find it impossible, *aim to make out a set of* neatly laid out, *coherent notes during the lecture*. Some people prefer to make rough notes and re-write them afterwards but very few can sustain the effort required for this. In any case, *the time after the lecture can be spent more profitably*.

After the lecture

As soon as possible, work through your notes, tidying up and expanding where necessary and checking the accuracy wherever there is any doubt (where, for instance, you may have copied something down in a rush). Try to learn any new definitions straightaway, make sure that you understand the details of any worked examples, pick out the important features of any new concepts and proofs. You may find it helpful to *extract the key points*, such as definitions, theorems, important examples or counter examples, etc. and write them on small filing cards (say 5" by 3"). Thread these together through a hole punched in the top left-hand corner and they will prove invaluable when it comes to revision time. An alternative method is to highlight the key points in your notes using one of the many fluorescent markers available. However you choose to do it, you should make sure that you make a good index or a summary for each set of notes.

So with pencil and paper at hand, *read your notes actively and annotate them* (possibly in a second colour).

Read your lecture notes, together with problem sheets, trying to sort material into order and into different categories; definitions, examples of definitions, techniques, minor theorems, major theorems, applications of theorems and techniques and finally, proofs of theorems. Look for examples both in your notes and in other sources (books, problem sheets, exam questions) and add these to your notes in the appropriate margin. Work together with other students in this. A syllabus will often help you clarify the structure of the module: if you have time, perhaps in the vacation, try to write out a summary of the module, but an overview of what you have done as you go along is invaluable.

If you have been told to read part of a textbook, or handout, do so as soon as you can while the subject is still reasonably fresh in the mind. Make notes on what you read—doing this is rather like working over your lecture notes and picking up the 'key points'. It is advisable to cross reference the material so that annotations in your lecture notes point to relevant parts of notes from books etc. and vice versa.

If you are having difficulty with the lecture content or the reading which your own efforts and discussions with other students cannot resolve, arrange to go to see your lecturer or tutor. Do this as soon as possible, otherwise you will fall behind and find yourself with several ill-understood lectures to sort out. *It will be easier for your lecturer to help if you can be precise about your difficulty*. Take your notes with you and be sure to have relevant places marked, with a concise note of your exact difficulty if possible. Don't rely on your memory or you may waste your own time and the lecturer's time.

Sometimes a change of topic in lectures will give you a fresh start but check to find out by when the difficulty must be resolved and don't let it 'ride'!

If all of the above advice seems rather daunting and hopelessly time-consuming, don't despair! Do what you can. You will find that you get quicker and better as time goes by. By following these ideas, you will acquire a record of each module which can be referred to easily at any stage, is a clear account of the topic, and is an invaluable aid at revision time.

OTHER WAYS OF LEARNING

Lectures aren't everything!

TUTORIALS

Make sure you

- come prepared
- have noted down points of difficulty
- participate constructively

PROBLEM SESSIONS

Make an effort to

- work in small groups
- ask for help
- attend regularly

TEXT BOOKS

Buy some, borrow some, share some

- buy your own copy of set texts
- browse in the library for alternatives
- use the index

EQUIPMENT

Technology can help you

- buy a good calculator
- PCs are provided, so only buy after taking soundings

LIBRARY/RESOURCES CENTRE

Use what's provided

- know where to look
- know how to use the catalogue
- know how to use different computers

OTHER WAYS OF LEARNING

Tutorials
Back-up to lectures varies from place to place, and sometimes from module to module, but often includes tutorials. These may be discussions with your tutor individually or in small groups. The format will be determined by the tutor but whatever it is you can help to get the best out of it by attending to three points, namely,

1. Make sure that you bring all relevant notes, paper, pen and calculator.

2. If there is work to be done before the tutorial do attempt it and take your efforts along with you, even if you haven't got very far.

3. *Get involved*! Don't be afraid to look silly by raising your real problems however trivial they may seem, the chances are that others share them. If you are really shy about asking questions yourself try to form a group and appoint a spokesperson.

Problem Sessions
Problem sessions are usually taken in larger groups than tutorials. Problems are worked by students during the session and help is on hand to get you over difficulties. You will probably find it helpful to work with one or two other students in the group.

The same comments 1, 2, and 3 as for tutorials apply. It is a good idea to attempt a range of problems of varying difficulty during the session so that you may gain full benefit from the help available.

Textbooks
These are often recommended by your lecturer or tutor to supplement or to replace lecture notes. *Having your own copy is a sensible plan, so that you can annotate it* and use it at revision time when the library copy is in heavy demand. It often pays to ask around whether there are any second hand books for sale in the department or bookshops, Student's Union, etc.

The most important aspect of reading is to be clear in your own mind why you are reading: to get a rapid *overview* of the topic in order to see what is relevant or not; to *search* for a specific piece of information: come to grips with and *understand* a particular theorem or proof! Each of these tasks requires a rather different reading strategy. *Having a clear purpose in mind when reading is one of the most important study skills that you need to develop*.

Books vary widely in the notation they use and in their approach to a subject. During the early stages you should stick to books (if any) recommended by the lecturer and books which use similar conventions and notations. A 'good' book is one which suits you but you will probably want one with:

1. Lots of worked examples
2. Lots of exercise and problems
3. Answers at the back
4. Attractive presentation

You cannot read a mathematical textbook like a novel — you will rarely want to read from cover to cover. Learn to use the contents page and the index to find the relevant sections and pages; sometimes you may have to skim through both to find where to look. Remember that you may need to consult a list of notations, conventions, and definitions either at the front of the book, the back, or where they first appear in the

main text. Read slowly and carefully and always with pen and paper to hand to do supplementary working. If there is a lot of text on a page, blanking some of it out may help concentration.

Be prepared to go over an argument several times. If you have to gloss over a step, note it for later consideration. As always, try to understand the overall structure of an argument as well as the detailed steps. Make notes and cross reference them with your other notes on the subject. Always indicate on these notes the book from which they were taken together with the page numbers so that you can find it again quickly if necessary.

Problems in the text often have hints (and even answers) at the end of the book; these may help you with work on problem sheets. Definitions and notations differ slightly from one book to another and from lecture notes; sometimes these differences matter, sometimes they don't — consult your tutor if in any doubt. There may be an index of symbols and notations to help you.

Coursework

Coursework can provide a substantial back up to your learning. It can often help to reinforce material already covered and, equally, can often provide a firm foundation on which to build new ideas and techniques.

Exploiting Technology — computers and calculators

Just as you now use a calculator as a standard tool for numerical and statistical calculations, you will find it beneficial to exploit the uses of computers and more sophisticated and hand-held calculators. If your old calculator is faulty, consider getting a graphical calculator to help you see function behaviour. However make sure you know which calculators you are allowed to take into examinations. Only invest in a PC after you know you really need it and have taken advice about which to buy.

Pictures and graphs can often make the difference between partial understanding and getting a real grip on mathematics. These can now be readily obtained using computers or graphical calculators. For example, plotting functions immediately shows a whole range of features including roots turning points, discontinuities etc. — an invaluable aid not only for completing some tutorial exercises but in real problem solving.

Find out what your computer laboratory offers:

1. word processing packages can help you organise your notes and prepare reports; spreadsheets may be useful in planning your time;

2. most computer laboratories have an extensive library of software available; ask your lecturer what is suitable for your module;

3. look out for Computer-Assisted Learning (CAL) software that you could use to reinforce your understanding of topics which you find difficult;

4. animations can clarify a variety of topics ranging from moving bodies in dynamics to the rotation of computer generated images;

5. computer software now exists that can perform algebraic manipulation thus taking the drudgery out of simple tasks like factorisation and more difficult tasks like solving algebraically large systems of equations or integrating analytically complicated functions;

6. most libraries contain software for popular numerical and statistical methods.

A word of warning—computers are not infallible; you should be aware that there are limitations, for example, in terms of accuracy, which occasionally cause problems. Don't be afraid to ask your lecturer if the unexpected happens. Ensure that you know where all the computer laboratories are and which ones you can use in your own time. They may not all be the same; make a note of the log-on sequence and any other peculiarities of each. If you have to use a package make sure that you have a user guide.

Library / Resources centre

Get to know your library well, it is the source of a vast amount of information. You will have had introductory talks and tours of the Science Library; but you may need to use other libraries (such as the Hallward Library) for some modules, so make yourself familiar with them also. Find out how to use the catalogue both the online computer catalogue (which does searches) and standard card-file ones, find out where the maths books are stacked and spend some time browsing to see what is there. It will repay you in the long term. Occasionally browsing may unexpectedly help you to resolve a difficulty or clarify a muddle.

The library staff are there to help you—don't be afraid to ask. There may be librarians with a special interest in and knowledge of mathematics books.

If there is a Resources Centre, find out where it is and what is available there. It may have film loops, tape-slide or tape-book presentations, video-tapes, programmed learning material etc., and someone to show you how to use all of these things. Your tutor or librarian may be able to help you with this. Second and third year students can tell you what they discovered and found useful.

REVISION TECHNIQUES

Mathematics is best learnt by doing!

PRACTISE EXAM TECHNIQUES

Have you acquired the skills of:

- *reading and analysing exam questions bit by bit*
  (Make sure you know precisely what is wanted)
- *looking for hints in the questions themselves*
  (Can you spot the hidden clues?)

PRACTISE EXAM QUESTIONS

Make sure you have:

- *an up-to-date syllabus*
  (Don't work up answers based on topics that have been dropped from the syllabus)
- *your lecture notes to hand*
  (But see how far you can go without recourse to your notes)
- familiar textbooks available for immediate reference
  (Ones that you know your way around)

USE YOUR TUTOR SENSIBLY

One useful approach is to:

- *make a list of specific points you don't understand (e.g. from notes, problem sheets, textbooks, etc.) and arrange to see your tutor to go through these with her/him.*

CO-OPERATE WITH FELLOW STUDENTS

Why not try

- *joint problem solving sessions*
- *analysing exam questions and comparing draft solutions together*

PREPARE A REVISION PLAN

Be flexible but try to keep broadly to your plan and:

- *be clear about what you hope to achieve from each revision session*
  (Don't be too ambitious)
- *identify likely topics from past papers, lecture notes, the syllabus and your tutor's hints*
  (Orientate your revision with these topics in mind)
- *look for links between topics*
  (It will help if you can see the course as a whole)
- *make sure you can reproduce proofs and solve straightforward exercises with ease*
  (Don't miss out on the 'easy' marks)
- *work hard but don't overdo it*
  (Often a problem for the most conscientious students)

REVISION TECHNIQUES

General Advice
Like it or not, one of your main preoccupations will be with exams. Time spent in *'sussing out' the demands of the exam system and orientating your work with these demands in mind* is time well spent. Lecturers differ widely in the 'hints' they give students about exams and the kind of work they are looking for e.g. the balance between 'bookwork' and problems. So get to know your lecturers, listen to what they have to say about exam papers. Be sure to talk about what is likely to 'come up' with your fellow students. Strangely, the most successful students are not those with the greatest knowledge and understanding of their discipline but rather those who are most adept at weighing up the system.

According to one dictionary, 'revision' means "renewed study with a view to refreshing and repairing gaps in the memory". It is not a substitute for steady, hard work whilst modules are in progress but the last stage of a work programme which has been proceeding throughout the whole year. The revision period before exams is a time for re-familiarising yourself with ideas which may have been crowded out by more recent work, rather than trying to understand new work.

As with every other aspect of study, you must find out what suits you personally but you may find some of the following advice useful.

Ideally, the process of revision begins when you first meet the material in lectures; the longer you spend getting to know it, the easier it is in the weeks before exams. Work out, with your tutor's help if necessary, what formulae and proofs you need to memorise and what can be worked out on the spot. Memorising should not be parrot fashion but by familiarity through regular use. Some formulae, however, will be familiar but hard to remember and these you should be able to derive where necessary from more basic principles.

A useful device for consolidating the module is the preparation of patterned or 'skeleton' notes compiled, perhaps, from the list of key areas and words with some indication of their inter-relationships. You should also ascertain the format of the individual exam papers from the lecturer and the topics to be assessed. Revise these areas efficiently, e.g. by means of your skeleton notes. We recommend that you learn the basic steps in proofs rather than try to commit the entire proof to memory.

A planned strategy
Work out a strategy and plan your campaign, for example:

1. *Mathematics is best revised, as well as learned, by doing it, in particular, by solving problems*. Use old exam papers (but check for syllabus changes) for 'dummy runs'; practice doing exam questions under self-imposed exam conditions. Ask the lecturer if he will hand out revision sheets of problems of the type that may appear in your exam. This is especially important if there are few, or no, past papers or if there has been a change in the module lecturer or syllabus.

2. *Prepare a clear plan for your revision session and try to stick to it…* Make a list of aims for each revision session and don't get sidetracked. You will probably find it helpful to divide each set of notes into topics, perhaps based on past papers or lecture headings, and to revise one topic at a time. However, you should also try to get an overall view of the module and look for links between topics.

3. Use your revision to *practise examination technique* too. Get into the habit of reading each question carefully, bit-by-bit and analysing it into its component parts so that you know exactly what is required in the answer. Write your solutions neatly, clearly and concisely. Speed of writing is not so important in mathematics as in essay-based subjects: clarity of thought and economy of expression are what matter. Try a few questions within the time limit which would be allowed for them in the exam (usually about half an hour). After the time allowed for the question is up, stop and assess what you have done and how it might be improved. You will probably be rather slow at first but as you practise doing questions on a given topic you will speed up. Some students find it helpful to keep one past paper for a final run through under exam conditions.

4. Difficulties which you encounter when trying to do problems and exam questions will force you back to your lecture notes for information on the topic you are revising. This is the right way to get into your notes—*you will get nowhere if you simply sit down and try to read them through from the beginning with the idea that you will try some problems when you have mastered the notes!* Now is the time when your summary of key-facts and index will prove invaluable in enabling you to locate the information you need quickly—but you still need to go to the notes themselves for details. Do not forget!—*always read with pen or pencil in hand*. Remember that you have not achieved mastery of a particular point until you can reproduce proofs etc. with understanding and without reference to your notes, and solve straightforward exercises quickly and with ease.

5. Try to make sure that you really do understand and are able to use the things you think you know. You should aim to be able to do the standard bookwork and routine parts of questions efficiently and without too much effort.

6. If you have difficulty in trying to cover the entire range of a module, concentrate on a smaller number of topics and revise them thoroughly. Give priority to those topics which appear frequently in past papers and those you feel most at home with. Remember that you are (probably!) not aiming at 100% coverage.

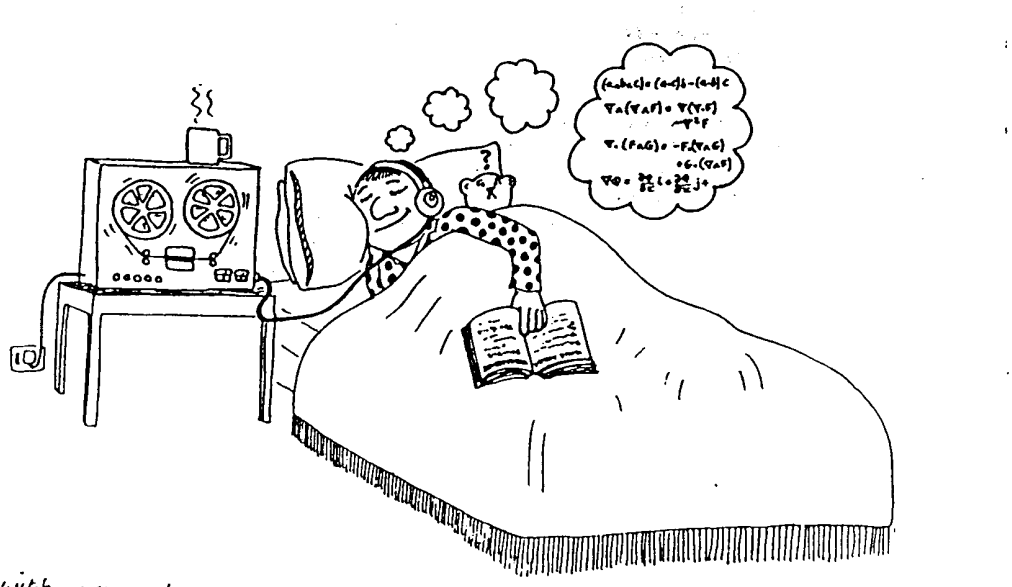
7. Every revision plan should pay special attention to the basic mathematical techniques of the module, especially those which have been used frequently and those from earlier modules which keep cropping up.

8. If you use books for revision it is best to stick to the ones you already know—the revision period is no time to start grappling with an unfamiliar approach or notation.

9. When going through your notes, homework sheets and past problems on a module, make a list of the points you do not understand and the problems you cannot do. Arrange to see the appropriate lecturer or tutor to go through your difficulties. Bear in mind that everyone else may be doing the same, so you will need to make a proper appointment and try to identify your difficulties as precisely as you can to make the best of the time s/he can give to you. Don't use this as a substitute for sorting out difficulties earlier in the year immediately after the lecture in which they arose.

10. *Work hard, but sensibly—don't overdo it*. Make sure you get adequate relaxation, exercise and sleep. It is usually not a good idea to revise for 14 hours a day! You can't keep your concentration going for such a length of time. A better method is to break the revision day up into five or six periods of about 90 minutes, punctuated by intervals in which you get some fresh air, have meals, etc. A total *working* day of 8 or 9 hours is as much as most students can cope with on a regular basis. For many students it is probably best to avoid working right up to the exam itself; otherwise you may cause your mind to become confused and actually impair your performance

in the exam. Some can benefit from looking up theorems, definitions, formulae, etc. at the last moment but others may do better by taking a complete break for the last couple of days. The important thing is to know what suits you best.



As with every other aspect of study, you must find out what suits you personally

WINNING AT EXAMINATIONS

BEFORE THE EXAM
Prepare yourself by:

- *checking that you have remembered everything you have been told about the paper*
  (time and place, rubric, mark up procedure…)

- *making a checklist of what you need to take*
  (pens, pencils, calculator…)

- *looking at past papers and noticing the form they take*
  (identify likely questions and mark distribution)

IN THE EXAM
Invigilators

- *invigilators are student friendly*

Strategy does matter. Make sure you:

- *stay cool and don't panic!*
- *don't rush into starting a question*
- *read the whole paper carefully to start with*
- *decide which questions you will attempt*
- *allocate your time among questions*

Start with a question you feel most confident about and:

- *read it carefully*
  (Answer the question set, not the one you wish had been set!)

- *analyse exactly what you are asked to do*
  (Distinguish between: define/explain/prove/show, etc.)

- *set out your answer legibly and clearly*
  (This helps you avoid making silly mistakes. After all, marks can only be given for work that can be read!)

- *if you do get stuck, explain in words what you were trying to do, assume the result and move on to the next part of the question*
  (After all, who knows everything?)

- *at the end, check you have answered the question that was asked and you have not left a section out*
  (But try not to leave the examination room early. Another few minutes checking your answers might make all the difference.)

AFTER THE EXAM

- *Forget it! Concentrate on the next paper.*

WINNING AT EXAMINATIONS

Before the exam
Make sure that you get hold of, and read, any regulations or other material which is available. Find out how your papers will be marked and what you are required to do in order to pass/do well. Things you will need to know include:

1. Are you allowed to take any handwritten materials and/or books into the examination?
2. What kind of calculator (if any) is allowed?
3. Is more credit given for complete answers?
4. Do all answers count, or only the best four or five, say?
5. Are there any printing conventions, e.g. are parts of questions labelled (a), (b), (c), if they are entirely separate and (i), (ii), (iii), if they are related ?

Make sure that you have checked the examination timetable accurately, that there are no clashes or inconsistencies and check just before the exam for any changes that might have been made.

Make a list of items you will need to take to the exam. This might include: pens, ink, pencils, eraser, ruler, drawing instruments, calculator (with spare batteries!) watch and exam number. If nothing else such a list will remove one source of anxiety on exam day.

If the examination is “open book” make sure that you arrange the materials you intend to take with you in a clear and sensible way. Indexing handwritten notes can be a considerable help.
Get to the exam room in good time, so that you have no last-minute panics if anything does delay you slightly. Your aim is to start the exam in as calm a mood as possible (try a few deep breaths!). Don’t take medicines or stimulants of any kind except under medical supervision. (At least, not until after the exams!)

In the exam
Remember that invigilators are not just there to supervise the exam, they are also there to help. If you think there is a mistake or ambiguity on the paper, ask. If you don’t feel well, or simply need a glass of water, say so.

When the exam starts, don’t be in a hurry to put pen to paper. *Strategy matters*. **Read the rubric carefully** to make sure what questions you have to answer and remember that it is *vitally important to do the correct number of questions*. Then read the questions themselves through. Note those you think you will be able to answer fairly well: make a rough allocation of your time among the questions you intend to answer; select the question you feel surest of, read it carefully, and begin but beware of spending more than your allocated time on it.

Analyse the whole question carefully. Note what information is given, exactly what you are required to do, whether the question is structured (with one part leading on to another), and whether any explicit hints are given (perhaps at the end of the question). Make sure you answer only the question as it is printed and do not include, for example, unnecessary proofs of results you are told you can use without proof, or examples to explain definitions. Often there are hints at the ends of questions: use them. Pay attention to words like ‘define’, ‘explain’, ‘prove’, ‘show’, ‘outline a proof of’; make sure you already know what these mean. Don’t write down a definition you’re not asked for unless you’re sure it will directly benefit you in answering the question; and then only do

it on rough paper. Check your arithmetic and solutions if possible, especially when later parts of the question depend on early parts—but not at the cost of omitting a question.

Be prepared to start a question in the middle if the first part seems intractable. When you get really stuck, explain in words what you were trying to do, then move on.

Work at your own speed: don't feel that you have to rush. Write legibly and lay out the work clearly so that you are not led into avoidable errors by misreading and misinterpreting your own work. Remember too, that you cannot get marks for work which the examiner cannot read or follow. *If you get stuck, don't panic!* You may find that one part of a question escapes you but that by assuming the result you can continue with the rest. Do so, and write a note to the examiner which makes it clear what you are doing. You may be able to complete the missing part later. Should you be completely stuck, don't keep going over and over the same ground again and again. Try the next question you intend doing and go back later if you have time to spare or a fresh idea occurs to you. *Always keep your time allocation in mind*.

After completing an answer, read the question again to make sure that you have answered the one that was asked and neither left a part out nor answered some slightly different question.

Sometimes, when you are working on one question ideas will occur to you for solving another. Make a rough note of them immediately for future reference so that you do not forget them.

When you have done as much of the paper as is required or as much as you feel able to do, don't leave. *If there is time left, sit back and relax for a few minutes and then think about the questions again.* You may see something that you missed the first time, or get a useful new idea. This can happen even quite late in the exam. If, on the other hand, *you are running out of time* on your last question, but know how you would have continued, *give a brief description of your intended method*.

At the end of the exam, be sure to hand in everything you have written.

After the exam

Forget it for the moment, and concentrate on the next paper. If you are subsequently asked to attend a viva, see the comment at the end of the next section.

WINNING AT COURSEWORK AND PROJECTS

SOME SUGGESTIONS

Getting started

- *Don't delay*

Getting on

- *talk to fellow students*
- *talk to staff*
- *make use of all possible facilities*
- *make good use of vacations*
- *don't overdo it*

Getting finished

- *finish in plenty of time*
- *present your work well*

Vivas

- *don't be frightened*

WINNING AT COURSEWORK AND PROJECTS

Getting started
Each piece of coursework will have a deadline. It may seem a long way off, but it isn't. The sooner you start the more time you'll have to think things out.

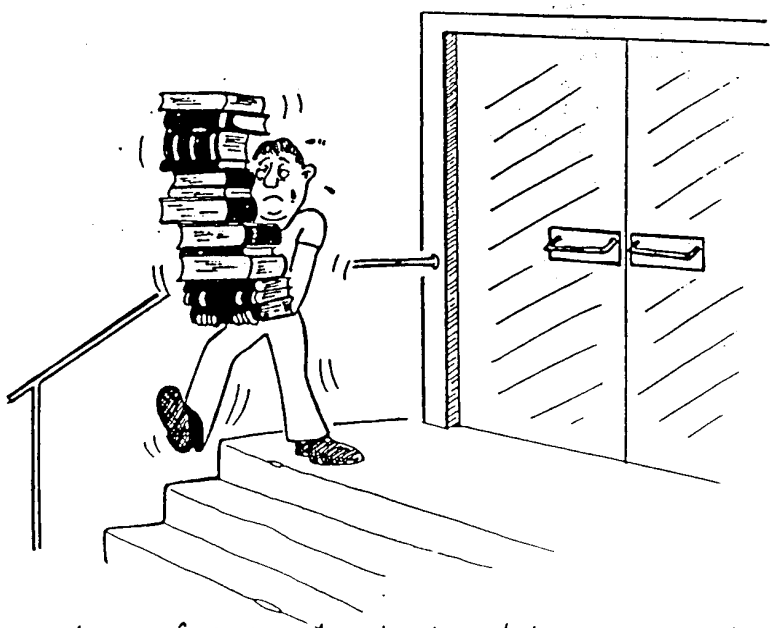
Getting on
There is no better way of developing ideas than by talking to others. That doesn't mean getting them to do all the work. Discussion stimulates your mind. Getting people to do the work for you is cheating.

Make good use of any materials which your tutor recommends you to study or, better, which you can find yourself. Books, computer software and suchlike can give you ideas. Make a careful note of any references for inclusion in your work.

It may be that the time you have to do the work includes a vacation. If so, make good use of it. Prepare yourself before you go and remember that a period of concentrated work is usually much more profitable than work continuously interrupted by other essential term-time activities.

Don't overdo it. Coursework and projects can 'take over'. Decide how much time you are going to allow yourself to finish and don't exceed it, otherwise other modules will suffer.

The 'writing-up' phase always takes longer than you expect and the *first draft* is unlikely to be the final version. But beware of spending time on lots of further iterations—the gain in marks may not be commensurate with the extra effort.



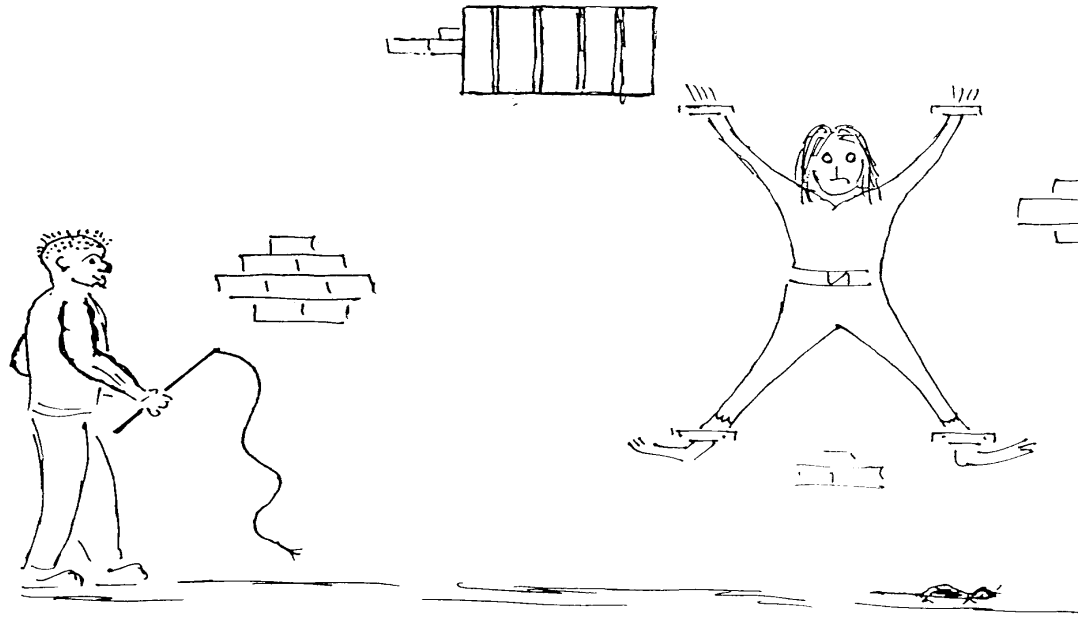
Make good use of any materials your tutor recommends...

Getting finished
The sooner you finish the better. If it is necessary to change things at the last moment there will be time; if not, you will experience an immense feeling of satisfaction. If the deadline is creeping up on you, you will become increasingly worried. Remember that for computer projects in particular, it is essential that your program works. Don't leave it until the last moment to try: not only will you find a failure frustrating but you will probably also have increasing difficulty in gaining access to equipment as the final date draws near.

Presentation is very important. Make good use of any word-processing or desktop publishing facilities available to you.

Late Submission
Assessed coursework or projects, which are submitted late, attract a standard penalty (5% absolute per working day), so it is crucial that you meet the deadline set. If unforeseen circumstances are likely to cause you to miss the deadline, then see the lecturer concerned, and your tutor, as soon as possible. You may be able to negotiate an extension, but only if you do so in advance of the deadline (and ideally not just the day before!).

Vivas
Sometimes you may be asked to attend a viva (oral examination). If you are, don't worry. The examiner is only trying to give you the opportunity to show you are worth more marks—s/he is not trying to demote you.



BOOKLIST

If you want more information about student study skills than we can include here then there are a number of books in the library and in the bookshop. Of those to be found in the Hallward library, we suggest:

Buzan, T., **Use your Head**, ISBN 0 563 208112, BBC Books 1989

Cassie, W., Fisher, and Constantine, T., **Student's Guide to Success**, ISBN 0 333 23277 1, Macmillan, 1977

Davis, P.J. and Hersh, Rueben, **The Mathematical Experience**, Brighton: Harvester Press, 1981

Gardner, M. – almost anything written by him

Howe, A. (1986) *How to Study: A Student's Guide to Effective Learning*, Great Britain: Kogan Page.

Kline, M., **Mathematics in a Western Culture**, Harmondsworth: Penguin, 1972

Lakatos, I., **Proofs and Refutations: the logic of mathematical discovery**, Cambridge: CUP (1976)

Marshall, A.M. and Rowland, F., **A Guide to Learning Independently**, Open University Press, 1989

Northedge, A., **The Good Study Guide**, The Open University, 1991

Polya, G., **How to Solve It**, Penguin Books, 1990

Reid, Jessie F., **A Guide to Effective Study**, ISBN 0 906093 00 7, Riverdale Press, 1997

Saunders, D. (1994) *The Complete Student Handbook*, Oxford: Blackwell Publishers