8

Flow in a water channel

A flowing stream of water where the flow has a free surface exposed to the open air is called a water channel. Included in the water channels, for example, are sewers. Roman waterworks were completed in 302BC with a water channel as long as 16.5 km. In AD305, 14 aqueducts were built with their water channels extending to 578 km in total, it is said. Anyway, water channels have a long history. Figures 1.1 and 8.1 show some remains.

Water channels have such large hydraulic mean depths that the Reynolds numbers are large too. Consequently the flow is turbulent. Furthermore, at such large Reynolds number, the friction coefficient becomes constant and is determined by the roughness of the wall.

Fig. 8.1 Remains of Roman aqueduct

8.1 Flow in an open channel with constant section and flow velocity

In an open channel, the flowing water has a free surface and flows by the action of gravity. As shown in Fig. 8.2, assume that water flows with constant

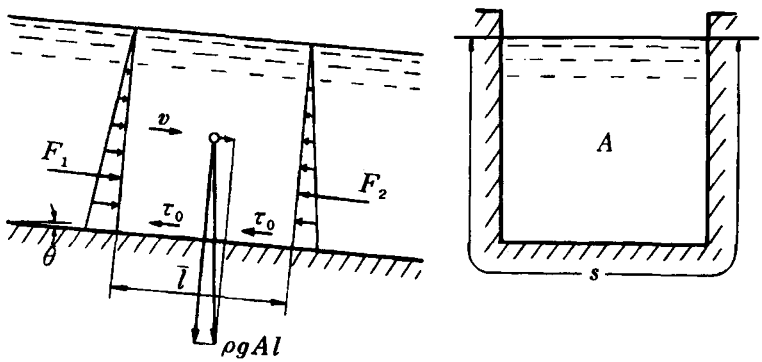


Fig. 8.2 Open channel

velocity v in an open channel of constant section and inclination angle θ of the bottom face. Now examine the balance of forces on water between the two sections a distance l apart. Since the water depth is uniform, the forces F_1 and F_2 acting on the sections due to hydrostatic pressure balance each other. Therefore, the only force acting in the direction of the flow is that component of water weight. Since the flow is not accelerating this force must equal the frictional force due to the wall. If the cross-sectional area of the open channel is A, the length of wetted perimeter s, and the mean value of wall shearing stress τ_0, then

$$\rho g A l \sin\theta = \tau_0 s l$$

Since θ is very small,

$$\text{inclination } i = \tan\theta \simeq \sin\theta$$

Then

$$\tau_0 = \rho g \frac{A}{s} i = \rho g m i \tag{8.1}$$

Here, $m = A/s$ is the hydraulic mean depth.

Expressing τ_0 as $\tau_0 = f\rho v^2/2$ using the frictional coefficient[1] f, then

$$v = \sqrt{\frac{2g}{f} mi} \tag{8.2}$$

Chezy expressed the velocity by the following equation as it was proportional to $\sqrt{mi}$:

$$v = c\sqrt{mi} \tag{8.3}$$

This equation is called Chezy's formula, with c the flow velocity coefficient. The value of c can be obtained using the Ganguillet–Kutter equation:

$$c = \frac{23 + 1/n + 0.00155/i}{1 + [23 + (0.00155/i)](n/\sqrt{m})} \tag{8.4}$$

[1] Note that $f = \lambda/4$ (see eqn (7.4)).

Table 8.1 Values of n in the Ganguillet–Kutter, the Manning and α in the Bazin equations

Wall surface condition	n	α
Smoothly shaved wooden board, smooth cement coated	0.010–0.013	0.06
Rough wooden board, relatively smooth concrete	0.012–0.018	
Brick, coated with mortar or like, ashlar masonry	0.013–0.017	0.46
Non-finished concrete	0.015–0.018	
Concrete with exposed gravel	0.016–0.020	1.30
Rough masonry	0.017–0.030	
Both sides stone-paved but bottom face irregular earth	0.028–0.035	
Deep, sand-bed river whose cross-sections are uniform	0.025–0.033	
Gravel-bed river whose cross-sections are uniform and whose banks are covered with wild grass	0.030–0.040	
Bending river with large stones and wild grass	0.035–0.050	2.0

It is also obtainable from the Bazin equation:

$$c = \frac{87}{1 + \alpha/\sqrt{m}} \tag{8.5}$$

More recently the Manning equation has often been used:

$$v = \frac{1}{n} m^{2/3} i^{1/2} \tag{8.6}$$

n in eqns (8.4) and (8.6) as well as α in eqn (8.5) are coefficients varying according to the wall condition. Their values are shown in Table 8.1. In general, the flow velocity is 0.5–3 m/s. These equations and the values appearing in Table 8.1 are for the case of SI units (units m, s).

The discharge of a water channel can be computed by the following equation:

$$Q = Av = Ac\sqrt{mi} = \frac{1}{n} Am^{2/3} i^{1/2} \tag{8.7}$$

The flow velocities at various points of the cross-section are not uniform. The largest flow velocity is found to be 10 ~ 40% of the depth below the water surface, while the mean flow velocity v is at 50 ~ 70% depth.

8.2 Best section shape of an open channel

If the section area A of the flow in an open channel is constant, and given that c and i in eqn (8.3) are also constant, if the section shape is properly selected so that the wetted perimeter is minimised, both the mean flow velocity v and the discharge Q become maximum.

Of all geometrical shapes, if fully charged, a circle has the shortest length of wetted perimeter for the given area. Consequently, a round water channel is important.

8.2.1 Circular water channel

Consider the relationship between water level, flow velocity and discharge for a round water channel of inner radius r (Fig. 8.3).

From eqns (8.6) and (8.7)

$$v = \frac{1}{n}\left(\frac{A}{s}\right)^{2/3} i^{1/2} \quad Q = \frac{1}{n}\frac{A^{5/3}}{s^{2/3}} i^{1/2}$$

$$A = r^2\left(\frac{\theta}{2}\right) - r^2\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right) = \frac{r^2(\theta - \sin\theta)}{2}$$

$$s = r\theta$$

$$m = \frac{r}{2}\left(1 - \frac{\sin\theta}{\theta}\right)$$

i.e.

$$v = \frac{1}{n} i^{1/2}\left[\frac{r}{2}\left(1 - \frac{\sin\theta}{\theta}\right)\right]^{2/3} \tag{8.8}$$

$$Q = \frac{1}{n} i^{1/2} \frac{\theta r^{8/3}}{2^{5/3}}\left(1 - \frac{\sin\theta}{\theta}\right)^{5/3} \tag{8.9}$$

Putting v_{full} and Q_{full} respectively as the flow velocity and the discharge whenever the maximum capacity of channel is flowing,

$$\frac{v}{v_{\text{full}}} = \left(1 - \frac{\sin\theta}{\theta}\right)^{2/3} \tag{8.10}$$

$$\frac{Q}{Q_{\text{full}}} = \frac{\theta}{2\pi}\left(1 - \frac{\sin\theta}{\theta}\right)^{5/3} \tag{8.11}$$

The relationship between θ and v, Q, is shown in Fig. 8.4.

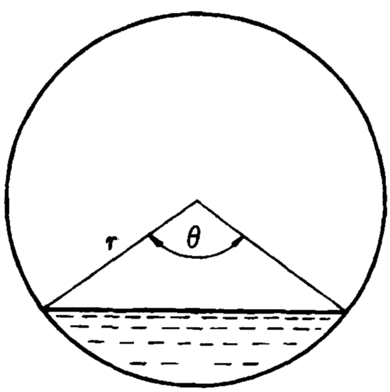


Fig. 8.3 Circular water channel

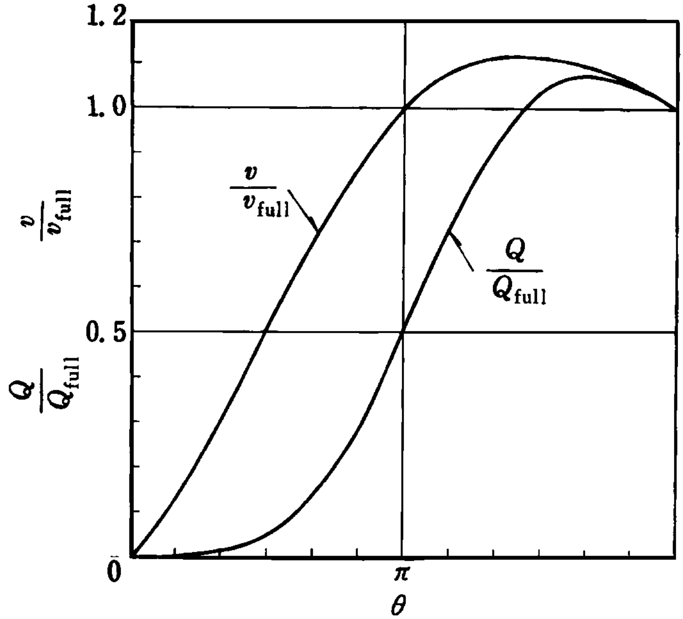


Fig. 8.4 Relationship between θ and v, Q

8.2.2 Rectangular water channel

For the case of Fig. 8.5, obtain the section shape where s is a minimum:

$$s = B + 2H = \frac{A}{H} + 2H$$

$$\frac{ds}{dH} = -\frac{A}{H^2} + 2 = 0$$

$$A = 2H^2$$

Therefore,

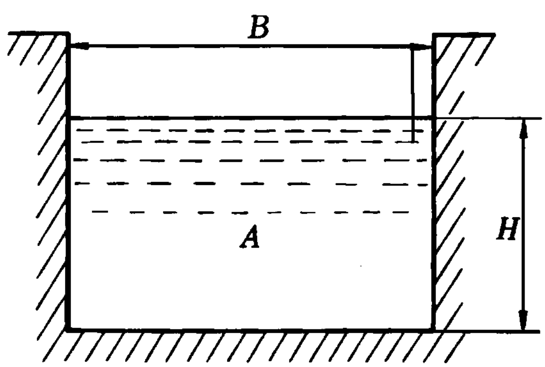


Fig. 8.5 Rectangular water channel

$$\frac{H}{B} = \frac{1}{2}$$

In other words, when c, A and i are constant, in order to maximise v and Q, the depth of the water channel should be one-half of the width.

8.3 Specific energy

Many open channel problems can be solved using the equation of energy. If the pressure is p at a point A in the open channel in Fig. 8.6, the total head of fluid at Point A is

$$\text{total head} = \frac{v^2}{2g} + \frac{p}{\rho g} + z + z_0$$

If the depth of water channel is h, then

$$h = \frac{p}{\rho g} + z$$

Consequently, the total head may be described as follows:

$$\text{total head} = \frac{v^2}{2g} + h + z_0 \tag{8.12}$$

However, the total head relative to the channel bottom is called the specific energy E, which expresses the energy per unit weight, and if the cross-sectional area of the open channel is A and the discharge Q, then

$$E = h + \frac{Q^2}{2gA^2} \tag{8.13}$$

This relationship is very important for analysing the flow in an open channel.

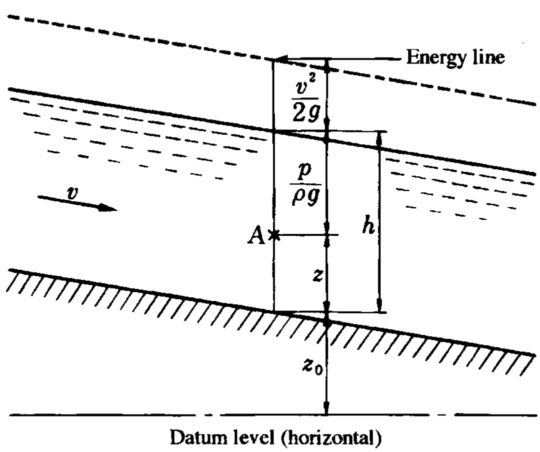


Fig. 8.6 Open channel

There are three variables, E, h, Q. Keeping one of them constant gives the relation between the other two.

8.4 Constant discharge

For constant discharge Q, the relation between the specific energy and the water depth is as shown in Fig. 8.7. The critical point of minimum energy occurs where $\mathrm{d}E/\mathrm{d}h = 0$.

$$\frac{\mathrm{d}E}{\mathrm{d}h} = 1 - \frac{Q^2}{gA^3}\frac{\mathrm{d}A}{\mathrm{d}h} = 0$$

Then

$$\frac{\mathrm{d}A}{\mathrm{d}h} = \frac{gA^3}{Q^2}$$

When the channel width at the free surface is B, $\mathrm{d}A = B\,\mathrm{d}h$. So the critical area A_c and the critical velocity v_c become as follows.

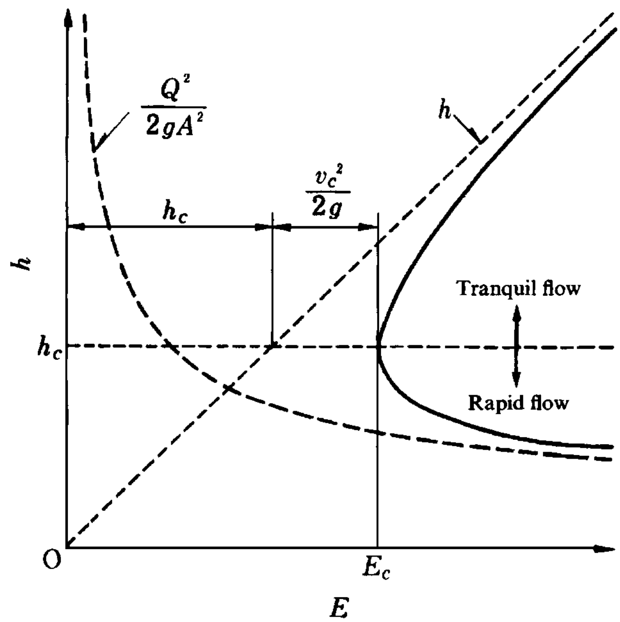


Fig. 8.7 Curve for constant discharge

$$\left.\begin{aligned} A_c &= \left(\frac{BQ^2}{g}\right)^{1/3} \\ v_c &= \frac{Q}{A_c} = \left(\frac{gA_c}{B}\right)^{1/2} \end{aligned}\right\} \tag{8.14}$$

Taking the rectangular water channel as an example, when the discharge per unit width is q, $Q = qB$. As the sectional area $A = hB$, the water depth h_c, eqn (8.15), which makes the specific energy minimum, is obtained from eqn (8.14).

$$h_c = \left(\frac{q^2}{g}\right)^{1/3} \tag{8.15}$$

At the critical water depth h_c,

$$E_c = \frac{q^2}{2gh_c^2} + h_c$$

From eqn (8.15)

$$q^2 = gh_c^3$$

$$E_c = \frac{h_c}{2} + h_c = 1.5h_c \tag{8.16}$$

The specific energy (total head) in the critical situation E_c is thus 1.5 times the critical water depth h_c. The corresponding critical velocity v_c becomes, as follows from eqn (8.15),

$$v_c = \frac{q}{h_c} = \sqrt{gh_c} \tag{8.17}$$

In the critical condition, the flow velocity coincides with the travelling velocity of a wave in a water channel of small depth, a so-called long wave.

If the flow depth is deeper or shallower than h_c, the flow behaviour is different. When the water is deeper than h_c, the velocity is smaller than the travelling velocity of the long wave and the flow is called tranquil (or subcritical) flow. When the water is shallower than h_c, the velocity is larger than the travelling velocity of the long wave and the flow is called rapid (or supercritical) flow.

8.5 Constant specific energy

For the case of the rectangular water channel, from eqn (8.13).

$$q^2 = 2g(h^2E - h^3)$$

$$\frac{dq}{dh} = \frac{g}{q}(2Eh - 3h^2) = 0$$

$$E_c = 1.5h_c \tag{8.18}$$

Comparing eqn (8.16) with (8.18), both the situation where the discharge is constant while the specific energy is minimum and that where the specific energy is constant while the discharge is maximum are found to be the same (Fig. 8.8).

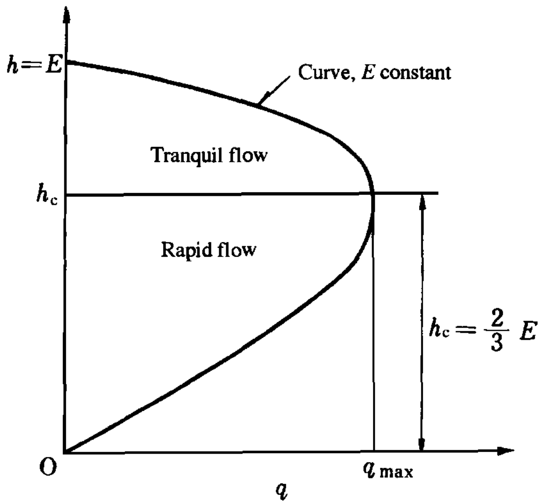


Fig. 8.8 Curve for constant specific energy

8.6 Constant water depth

For the case of the rectangular water channel, from eqn (8.13).

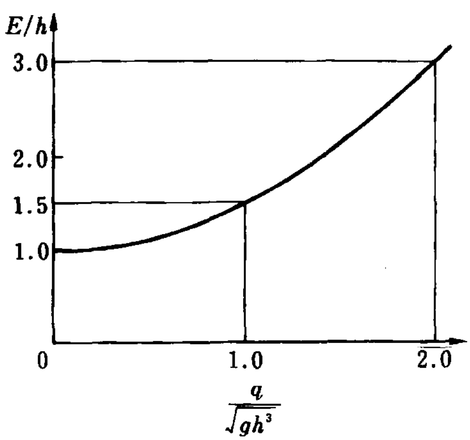


Fig. 8.9 Curve for constant water depth

$$\frac{E}{h} = 1 + \frac{q^2}{2gh^3} \tag{8.19}$$

The relationship between $q/\sqrt{gh^3}$ and E/h is plotted in Fig. 8.9. In words, the specific energy increases parabolically from 1 with q and, when the water depth is critical, i.e. $q^2 = gh^3$, $E/h = 1.5$.

8.7 Hydraulic jump

Rapid flow is unstable, and if decelerated it suddenly shifts to tranquil flow. This phenomenon is called hydraulic jump. For example, as shown in Fig. 8.10(a), when the inclination of a dam bottom is steep, the flow is rapid. When the inclination becomes gentle downstream, the flow is unable to maintain rapid flow and suddenly shifts to tranquil flow. A photograph of this situation is shown in Fig. 8.11.

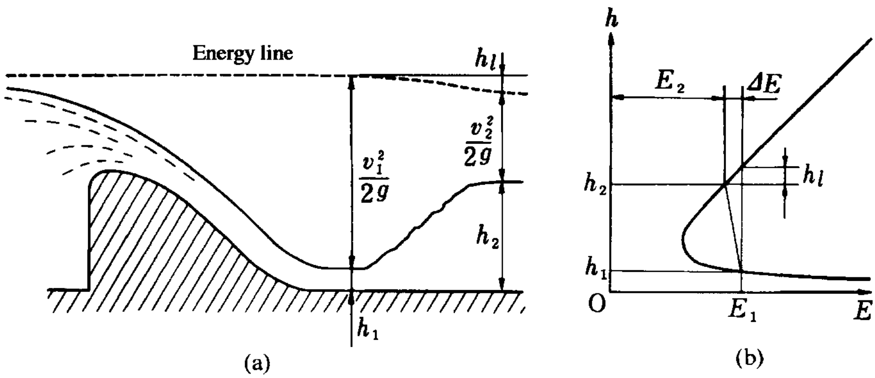


Fig. 8.10 Hydraulic jump

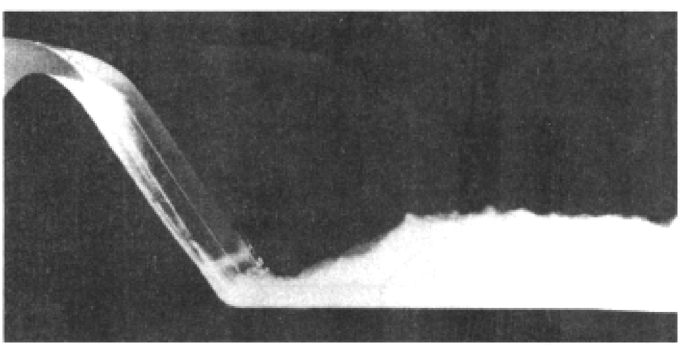


Fig. 8.11 Rapid flow and hydraulic jump on a dam

The travelling velocity a of a long wave in a water channel of small depth h is $\sqrt{gh}$. The ratio of the flow velocity to the wave velocity is called the Froude number. The Froude number of a tranquil flow is less than one, i.e. the flow velocity is smaller than the wave velocity. On the other hand, the Froude number of a rapid flow is larger than one; in other words, the flow velocity is larger than the wave velocity. Thus, tranquil flow and rapid flow in a water channel correspond to subsonic and supersonic flow, respectively, of a compressible gas.

For the flow of gas in a convergent–divergent nozzle (see Section 13.5.3), supersonic flow which has gone through the nozzle stays supersonic if the back pressure is low. If the back pressure is high, however, the flow suddenly shifts to the subsonic flow with a shock wave. In other words, there is an analogy between the hydraulic jump and the shock wave.

When a hydraulic jump is brought about, energy is dissipated by it (Fig. 8.10(b)). Thus erosion of the channel bottom further downstream can be prevented.

8.8 Problems

1. It is desired to obtain $0.5\,\mathrm{m^3/s}$ water discharge using a wooden open channel with a rectangular section as shown in Fig. 8.12. Find the necessary inclination using the Manning equation with $n = 0.01$.

2. For a concrete-coated water channel with the cross-section shown in Fig. 8.13, compare the discharge when the channel inclination is 0.002 obtained by the Chezy and the Manning equation. Assume $n = 0.016$.

3. Find the discharge in a smooth cement-coated rectangular channel 5 m wide, water depth 2 m and inclination 1/2000 using the Bazin equation.

4. Water is sent along the circular conduit in Fig. 8.14. What is the angle θ and depth h which maximise the flow velocity and the discharge if the radius $r = 1.5\,\mathrm{m}$?

5. In an open channel with a rectangular section 3 m wide, $15\,\mathrm{m^3/s}$ of water is flowing at 1.2 m depth. Is the flow rapid or tranquil and what is the specific energy?

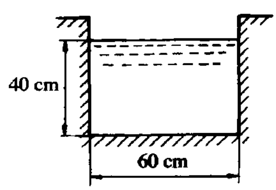


Fig. 8.12

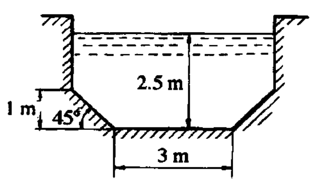


Fig. 8.13

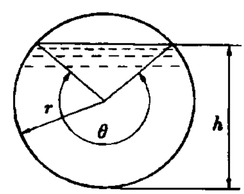
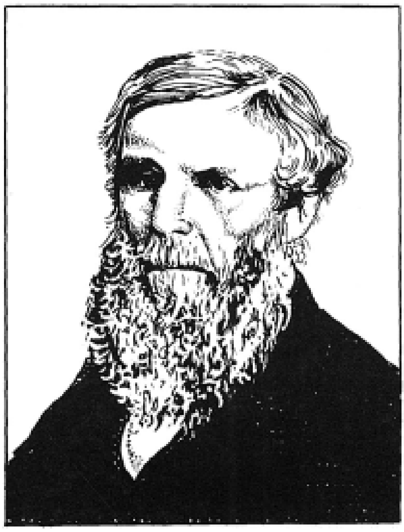


Fig. 8.14



William Froude (1810–79)
Born in England and engaged in shipbuilding. In his sixties started the study of ship resistance, building a boat testing pool (approximately 75 m long) near his home. After his death, this study was continued by his son, Robert Edmund Froude (1846–1924). For similarity under conditions of inertial and gravitational forces, the non-dimensional number used carries his name.

6. Find the critical water depth and the critical velocity when $12\,m^3/s$ of water is flowing in an open channel with a rectangular section 4 m wide.
7. What is the maximum discharge for 2 m specific energy in an open channel with a rectangular section 5 m wide?
8. Water is flowing at $20\,m^3/s$ in a rectangular channel 5 m wide. Find the downstream water depth necessary to cause this flow to jump to tranquil flow.
9. In what circumstances do the phenomena of rapid flow and hydraulic jump occur?