CHAPTER

25

Brakes

25.1 Introduction

A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc. The energy absorbed by brakes is dissipated in the form of heat. This heat is dissipated in the surrounding air (or water which is circulated through the passages in the brake drum) so that excessive heating of the brake lining does not take place. The design or capacity of a brake depends upon the following factors :

1. The unit pressure between the braking surfaces,
2. The coefficient of friction between the braking surfaces,
3. The peripheral velocity of the brake drum,

4. The projected area of the friction surfaces, and
5. The ability of the brake to dissipate heat equivalent to the energy being absorbed.

The major functional difference between a clutch and a brake is that a clutch is used to keep the driving and driven member moving together, whereas brakes are used to stop a moving member or to control its speed.

25.2 Energy Absorbed by a Brake

The energy absorbed by a brake depends upon the type of motion of the moving body. The motion of a body may be either pure translation or pure rotation or a combination of both translation and rotation. The energy corresponding to these motions is kinetic energy. Let us consider these motions as follows :

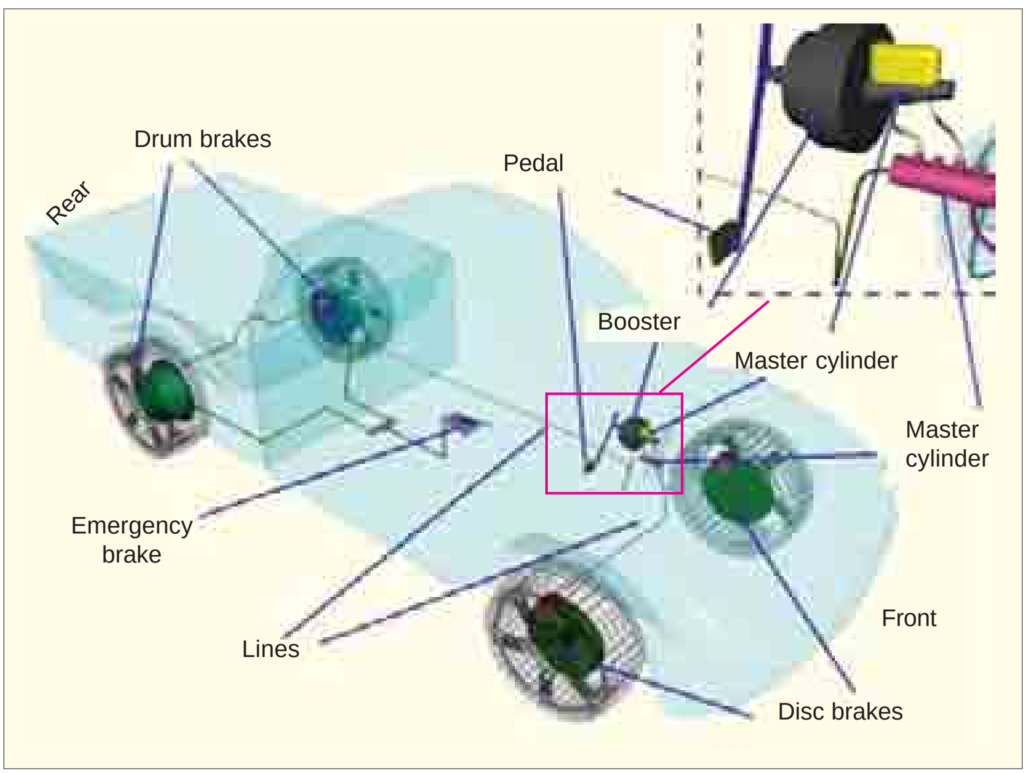
1. ***When the motion of the body is pure translation.*** Consider a body of mass (m) moving with a velocity v_1 m / s. Let its velocity is reduced to v_2 m / s by applying the brake. Therefore, the change in kinetic energy of the translating body or kinetic energy of translation,

$$E_1 = \frac{1}{2} m \left[(v_1)^2 - (v_2)^2 \right]$$

This energy must be absorbed by the brake. If the moving body is stopped after applying the brakes, then $v_2 = 0$, and

$$E_1 = \frac{1}{2} m (v_1)^2$$

2. ***When the motion of the body is pure rotation.*** Consider a body of mass moment of inertia I (about a given axis) is rotating about that axis with an angular velocity ω_1 rad / s. Let its angular velocity is reduced to ω_2 rad / s after applying the brake. Therefore, the change in kinetic energy of



Brake System Components

Note : This picture is given as additional information and is not a direct example of the current chapter.

Contents

the rotating body or kinetic energy of rotation,

$$E_2 = \frac{1}{2} I \left[(\omega_1)^2 - (\omega_2)^2 \right]$$

This energy must be absorbed by the brake. If the rotating body is stopped after applying the brakes, then $\omega_2 = 0$, and

$$E_2 = \frac{1}{2} I \, (\omega_1)^2$$

3. ***When the motion of the body is a combination of translation and rotation.*** Consider a body having both linear and angular motions, *e.g.* in the locomotive driving wheels and wheels of a moving car. In such cases, the total kinetic energy of the body is equal to the sum of the kinetic energies of translation and rotation.

∴ Total kinetic energy to be absorbed by the brake,

$$E = E_1 + E_2$$

Sometimes, the brake has to absorb the potential energy given up by objects being lowered by hoists, elevators etc. Consider a body of mass m is being lowered from a height h_1 to h_2 by applying the brake. Therefore the change in potential energy,

$$E_3 = m.g\,(h_1 - h_2)$$

If v_1 and v_2 m / s are the velocities of the mass before and after the brake is applied, then the change in potential energy is given by

$$E_3 = m.g \left(\frac{v_1 + v_2}{2} \right) t = m.g.v.t$$

where v = Mean velocity = $\dfrac{v_1 + v_2}{2}$, and

t = Time of brake application.

Thus, the total energy to be absorbed by the brake,

$$E = E_1 + E_2 + E_3$$

Let F_t = Tangential braking force or frictional force acting tangentially at the contact surface of the brake drum,

d = Diameter of the brake drum,

N_1 = Speed of the brake drum before the brake is applied,

N_2 = Speed of the brake drum after the brake is applied, and

N = Mean speed of the brake drum = $\dfrac{N_1 + N_2}{2}$

We know that the work done by the braking or frictional force in time t seconds

$$= F_t \times \pi \; d\,N \times t$$

Since the total energy to be absorbed by the brake must be equal to the wordone by the frictional force, therefore

$$E = F_t \times \pi\, d\, N \times t \qquad \text{or} \qquad F_t = \frac{E}{\pi\, d\, N.t}$$

The magnitude of F_t depends upon the final velocity (v_2) and on the braking time (t). Its value is maximum when $v_2 = 0$, *i.e.* when the load comes to rest finally.

We know that the torque which must be absorbed by the brake,

$$T = F_t \times r = F_t \times \frac{d}{2}$$

where r = Radius of the brake drum.

Top

25.3 Heat to be Dissipated during Braking

The energy absorbed by the brake and transformed into heat must be dissipated to the surrounding air in order to avoid excessive temperature rise of the brake lining. The *temperature rise depends upon the mass of the brake drum, the braking time and the heat dissipation capacity of the brake. The highest permissible temperatures recommended for different brake lining materials are given as follows :

1. For leather, fibre and wood facing = 65 – 70°C
2. For asbestos and metal surfaces that are slightly lubricated = 90 – 105°C
3. For automobile brakes with asbestos block lining = 180 – 225°C

Since the energy absorbed (or heat generated) and the rate of wear of the brake lining at a particular speed are dependent on the normal pressure between the braking surfaces, therefore it is an important factor in the design of brakes. The permissible normal pressure between the braking surfaces depends upon the material of the brake lining, the coefficient of friction and the maximum rate at which the energy is to be absorbed. The energy absorbed or the heat generated is given by

$$E = H_g = \mu . R_N . v = \mu . p . A . v \text{ (in J/s or watts)} \qquad \ldots(i)$$

where μ = Coefficient of friction,

R_N = Normal force acting at the contact surfaces, in newtons,

p = Normal pressure between the braking surfaces in N/m^2,

A = Projected area of the contact surfaces in m^2, and

v = Peripheral velocity of the brake drum in m/s.

The heat generated may also be obtained by considering the amount of kinetic or potential energies which is being absorbed. In other words,

$$H_g = E_K + E_P$$

where E_K = Total kinetic energy absorbed, and

E_P = Total potential energy absorbed.

The heat dissipated (H_d) may be estimated by

$$H_d = C\,(t_1 - t_2)\,A_r \qquad \ldots(ii)$$

where C = Heat dissipation factor or coefficient of heat transfer in W /m^2 / °C

$t_1 - t_2$ = Temperature difference between the exposed radiating surface and the surrounding air in °C, and

A_r = Area of radiating surface in m^2.

The value of C may be of the order of 29.5 W / m^2 /°C for a temperature difference of 40°C and increase up to 44 W/m^2/°C for a temperature difference of 200°C.

The expressions for the heat dissipated are quite approximate and should serve only as an indication of the capacity of the brake to dissipate heat. The exact performance of the brake should be determined by test.

It has been found that 10 to 25 per cent of the heat generated is immediately dissipated to the surrounding air while the remaining heat is absorbed by the brake drum causing its temperature to rise. The rise in temperature of the brake drum is given by

$$\Delta t = \frac{H_g}{m.c} \qquad \ldots(iii)$$

where Δt = Temperature rise of the brake drum in °C,

* When the temperature increases, the coefficient of friction decreases which adversely affect the torque capacity of the brake. At high temperature, there is a rapid wear of friction lining, which reduces the life of lining. Therefore, the temperature rise should be kept within the permissible range.

Contents

H_g = Heat generated by the brake in joules,

m = Mass of the brake drum in kg, and

c = Specific heat for the material of the brake drum in J/kg °C.

In brakes, it is very difficult to precisely calculate the temperature rise. In preliminary design analysis, the product *p.v* is considered in place of temperature rise. The experience has also shown that if the product *p.v* is high, the rate of wear of brake lining will be high and the brake life will be low. Thus the value of *p.v* should be lower than the upper limit value for the brake lining to have reasonable wear life. The following table shows the recommended values of *p.v* as suggested by various designers for different types of service.

Table 25.1. Recommended values of *p.v*.

S.No.	*Type of service*	*Recommended value of p.v in N-m/m² of projected area per second*
1.	Continuous application of load as in lowering operations and poor dissipation of heat.	0.98×10^6
2.	Intermittent application of load with comparatively long periods of rest and poor dissipation of heat.	1.93×10^6
3.	For continuous application of load and good dissipation of heat as in an oil bath.	2.9×10^6

Example 25.1. *A vehicle of mass 1200 kg is moving down the hill at a slope of 1: 5 at 72 km / h. It is to be stopped in a distance of 50 m. If the diameter of the tyre is 600 mm, determine the average braking torque to be applied to stop the vehicle, neglecting all the frictional energy except for the brake. If the friction energy is momentarily stored in a 20 kg cast iron brake drum, What is average temperature rise of the drum? The specific heat for cast iron may be taken as 520 J / kg°C.*

Determine, also, the minimum coefficient of friction between the tyres and the road in order that the wheels do not skid, assuming that the weight is equally distributed among all the four wheels.

Solution. Given : m = 1200 kg ; Slope = 1: 5 ; v = 72 km / h = 20 m/s ; h = 50 m ; d = 600 mm or r = 300 mm = 0.3 m ; m_b = 20 kg ; c = 520 J / kg°C

Average braking torque to be applied to stop the vehicle

We know that kinetic energy of the vehicle,

$$E_K = \frac{1}{2} m.v^2 = \frac{1}{2} \times 1200\,(20)^2 = 240\ 000 \text{ N-m}$$

and potential energy of the vehicle,

$$E_P = m.g.h \times \text{Slope} = 1200 \times 9.81 \times 50 \times \frac{1}{5} = 117\ 720 \text{ N-m}$$

∴ Total energy of the vehicle or the energy to be absorbed by the brake,

$$E = E_K + E_P = 240\ 000 + 117\ 720 = 357\ 720 \text{ N-m}$$

Since the vehicle is to be stopped in a distance of 50 m, therefore tangential braking force required,

$$F_t = 357\ 720 / 50 = 7154.4 \text{ N}$$

We know that average braking torque to be applied to stop the vehicle,

$$T_B = F_t \times r = 7154.4 \times 0.3 = 2146.32 \text{ N-m} \quad \textbf{Ans.}$$

Top

Average temperature rise of the drum

Let Δt = Average temperature rise of the drum in °C.

We know that the heat absorbed by the brake drum,

H_g = Energy absorbed by the brake drum

= 357 720 N-m = 357 720 J ... (∵ 1 N-m = 1 J)

We also know that the heat absorbed by the brake drum (H_g),

$$357\ 720 = m_b \times c \times \Delta t = 20 \times 520 \times \Delta t = 10\ 400\ \Delta t$$

$$\therefore \quad \Delta t = 357\ 720 / 10\ 400 = 34.4 °\text{C}$$ **Ans.**

Minimum coefficient of friction between the tyre and road

Let μ = Minimum coefficient of friction between the tyre and road, and

R_N = Normal force between the contact surface. This is equal to weight of the vehicle

$= m.g = 1200 \times 9.81 = 11\ 772$ N

We know that tangential braking force (F_t),

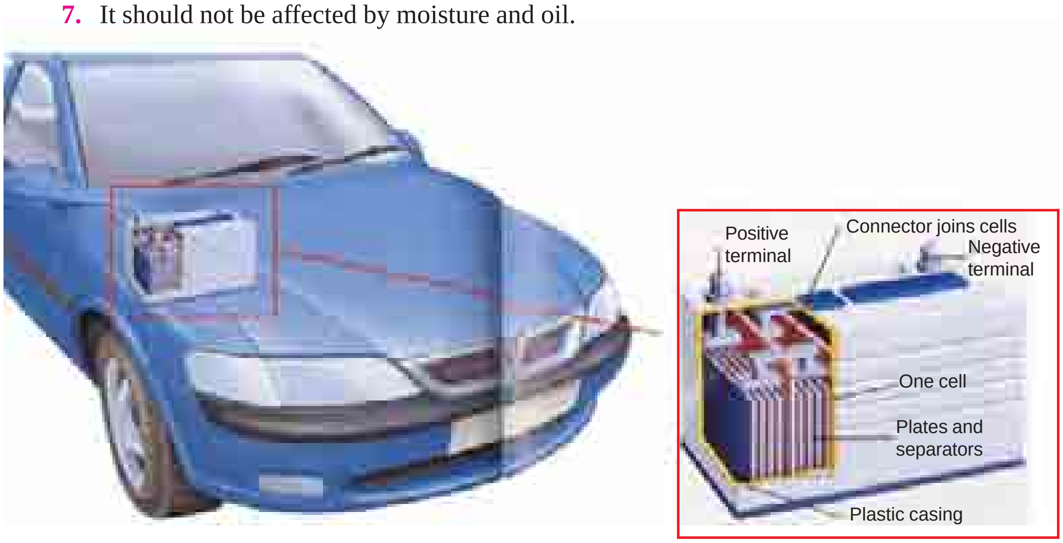
$$7154.4 = \mu.R_N = \mu \times 11\ 772$$

$$\therefore \quad \mu = 7154.4 / 11772 = 0.6$$ **Ans.**

25.4 Materials for Brake Lining

The material used for the brake lining should have the following characteristics :

1. It should have high coefficient of friction with minimum fading. In other words, the coefficient of friction should remain constant over the entire surface with change in temperature.
2. It should have low wear rate.
3. It should have high heat resistance.
4. It should have high heat dissipation capacity.
5. It should have low coefficient of thermal expansion.
6. It should have adequate mechanical strength.
7. It should not be affected by moisture and oil.



The rechargeable battery found in most cars is a combination of lead acid cells. A small dynamo, driven by the vehicle's engine, charges the battery whenever the engine is running.

Contents

The materials commonly used for facing or lining of brakes and their properties are shown in the following table.

Table 25.2. Properties of materials for brake lining.

Material for braking lining	*Coefficient of friction* (μ)			*Allowable pressure (p)*
	Dry	*Greasy*	*Lubricated*	*N/mm*^{2}
Cast iron on cast iron	0.15 – 0.2	0.06 – 0.10	0.05 – 0.10	1.0 – 1.75
Bronze on cast iron	–	0.05 – 0.10	0.05 – 0.10	0.56 – 0.84
Steel on cast iron	0.20 – 0.30	0.07 – 0.12	0.06 – 0.10	0.84 – 1.4
Wood on cast iron	0.20 – 0.35	0.08 – 0.12	–	0.40 – 0.62
Fibre on metal	–	0.10 – 0.20	–	0.07 – 0.28
Cork on metal	0.35	0.25 – 0.30	0.22 – 0.25	0.05 – 0.10
Leather on metal	0.3 – 0.5	0.15 – 0.20	0.12 – 0.15	0.07 – 0.28
Wire asbestos on metal	0.35 – 0.5	0.25 – 0.30	0.20 – 0.25	0.20 – 0.55
Asbestos blocks on metal	0.40 – 0.48	0.25 – 0.30	–	0.28 – 1.1
Asbestos on metal (Short action)	–	–	0.20 – 0.25	1.4 – 2.1
Metal on cast iron (Short action)	–	–	0.05 – 0.10	1.4 – 2.1

25.5 Types of Brakes

The brakes, according to the means used for transforming the energy by the braking element, are classified as :

1. Hydraulic brakes *e.g.* pumps or hydrodynamic brake and fluid agitator,
2. Electric brakes *e.g.* generators and eddy current brakes, and
3. Mechanical brakes.

Shoes of disk brakes of a racing car

The hydraulic and electric brakes cannot bring the member to rest and are mostly used where large amounts of energy are to be transformed while the brake is retarding the load such as in laboratory dynamometers, high way trucks and electric locomotives. These brakes are also used for retarding or controlling the speed of a vehicle for down-hill travel.

The mechanical brakes, according to the direction of acting force, may be divided into the following two groups :

Top

Contents

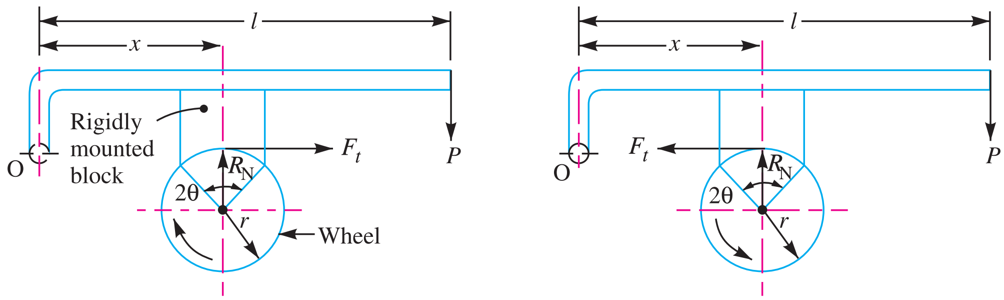
(*a*) ***Radial brakes.*** In these brakes, the force acting on the brake drum is in radial direction. The radial brakes may be sub-divided into ***external brakes*** and ***internal brakes.*** According to the shape of the friction element, these brakes may be block or shoe brakes and band brakes.

(*b*) ***Axial brakes.*** In these brakes, the force acting on the brake drum is in axial direction. The axial brakes may be disc brakes and cone brakes. The analysis of these brakes is similar to clutches.

Since we are concerned with only mechanical brakes, therefore, these are discussed in detail, in the following pages.

25.6 Single Block or Shoe Brake

A single block or shoe brake is shown in Fig. 25.1. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material than



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

Fig. 25.1. Single block brake. Line of action of tangential force passes through the fulcrum of the lever.

the rim of the wheel. This type of a brake is commonly used on railway trains and tram cars. The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed as shown in Fig. 25.1. The other end of the lever is pivoted on a fixed fulcrum *O*.

Let P = Force applied at the end of the lever,

R_N = Normal force pressing the brake block on the wheel,

r = Radius of the wheel,

2θ = Angle of contact surface of the block,

μ = Coefficient of friction, and

F_t = Tangential braking force or the frictional force acting at the contact surface of the block and the wheel.

If the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel,

$$F_t = \mu . R_N \quad \text{...(i)}$$

and the braking torque, $$T_B = F_t . r = \mu \, R_N . r \quad \text{...(ii)}$$

Let us now consider the following three cases :

Case 1. When the line of action of tangential braking force (F_t) passes through the fulcrum *O* of the lever, and the brake wheel rotates clockwise as shown in Fig. 25.1 (*a*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N \times x = P \times l \qquad \text{or} \qquad R_N = \frac{P \times l}{x}$$

Top

Contents

$\therefore$ Braking torque,
$$T_B = \mu.R_N.r = \mu \times \frac{P.l}{x} \times r = \frac{\mu.P.l.r}{x}$$

It may be noted that when the brake wheel rotates anticlockwise as shown in Fig. 25.1 (*b*), then the braking torque is same, *i.e.*

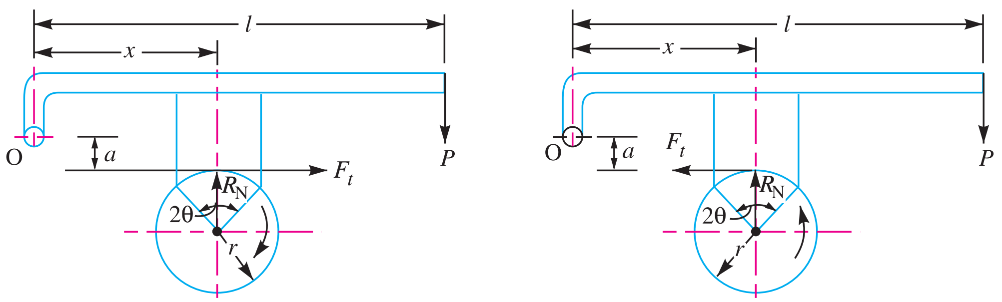
$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x}$$

Case 2. When the line of action of the tangential braking force (F_t) passes through a distance '*a*' below the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig. 25.2 (*a*), then for equilibrium, taking moments about the fulcrum *O*,

$$R_N \times x + F_t \times a = P.l$$

or
$$R_N \times x + \mu\, R_N \times a = P.l \qquad \text{or} \qquad R_N = \frac{P.l}{x + \mu.a}$$

and braking torque,
$$T_B = \mu\, R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$$



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

Fig. 25.2. Single block brake. Line of action of F_t passes below the fulcrum.

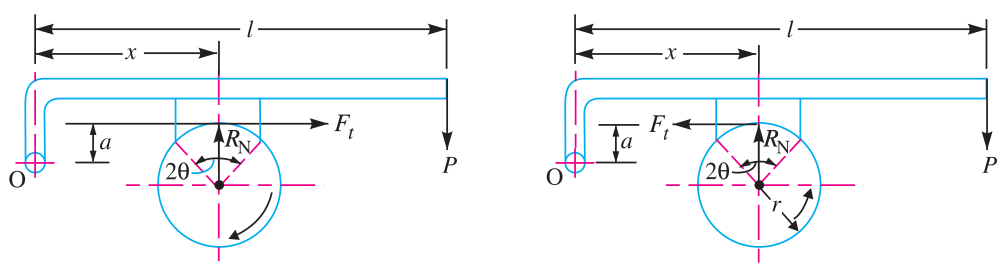
When the brake wheel rotates anticlockwise, as shown in Fig. 25.2 (*b*), then for equilibrium,

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a \qquad \text{...(i)}$$

or
$$R_N\,(x - \mu.a) = P.l \qquad \text{or} \qquad R_N = \frac{P.l}{x - \mu.a}$$

and braking torque,
$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

Case 3. When the line of action of the tangential braking force passes through a distance '*a*' above the fulcrum, and the brake wheel rotates clockwise as shown in Fig. 25.3 (*a*), then for equilibrium, taking moments about the fulcrum *O*, we have



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

Fig. 25.3. Single block brake. Line of action of F_t passes above the fulcrum.

Top

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a \qquad ...(ii)$$

or $$R_N(x - \mu.a) = P.l \quad \text{or} \quad R_N = \frac{P.l}{x - \mu.a}$$

and braking torque, $$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

When the brake wheel rotates anticlockwise as shown in Fig. 25.3 (*b*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N \times x + F_t \times a = P.l$$

or $$R_N \times x + \mu.R_N \times a = P.l \quad \text{or} \quad R_N = \frac{P.l}{x + \mu.a}$$

and braking torque, $$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$$

Notes: 1. From above we see that when the brake wheel rotates anticlockwise in case 2 [Fig. 25.2 (*b*)] and when it rotates clockwise in case 3 [Fig. 25.3 (*a*)], the equations ***(i)*** and ***(ii)*** are same, *i.e.*

$$R_N \times x = P.l + \mu.R_N.a$$

From this we see that the moment of frictional force ($\mu.R_N.a$) adds to the moment of force (*P.l*). In other words, the frictional force helps to apply the brake. Such type of brakes are said to be **self energizing brakes.** When the frictional force is great enough to apply the brake with no external force, then the brake is said to be **self-locking brake.**

From the above expression, we see that if $x \le \mu.a$, then *P* will be negative or equal to zero. This means no external force is needed to apply the brake and hence the brake is self locking. Therefore the condition for the brake to be self locking is

$$x \le \mu.a$$

The self-locking brake is used only in back-stop applications.

Shoe of a bicycle

2. The brake should be self-energizing and not the self-locking.

3. In order to avoid self-locking and to prevent the brake from grabbing, *x* is kept greater than $\mu.a$.

4. If A_b is the projected bearing area of the block or shoe, then the bearing pressure on the shoe,

$$p_b = R_N / A_b$$

We know that A_b = Width of shoe × Projected length of shoe = $w(2r \sin \theta)$

5. When a single block or shoe brake is applied to a rolling wheel, an additional load is thrown on the shaft bearings due to heavy normal force (R_N) and produces bending of the shaft. In order to overcome this drawback, a double block or shoe brake, as discussed in Art. 25.8, is used.

25.7 Pivoted Block or Shoe Brake

We have discussed in the previous article that when the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. But when the angle of contact is greater than 60°, then the unit pressure normal to the surface of contact is less at the ends than at the centre. In such cases, the block or shoe is pivoted to the lever as shown in Fig. 25.4, instead of being rigidly attached to the lever. This gives uniform wear of the brake lining in the direction of the applied force. The braking torque for a pivoted block or shoe brake (*i.e.* when $2\theta > 60°$) is given by

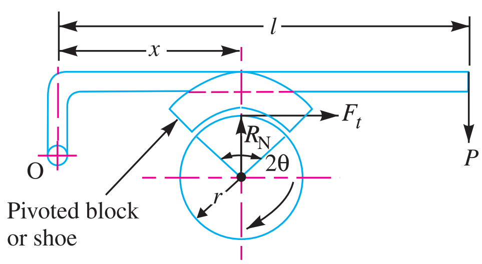
Contents

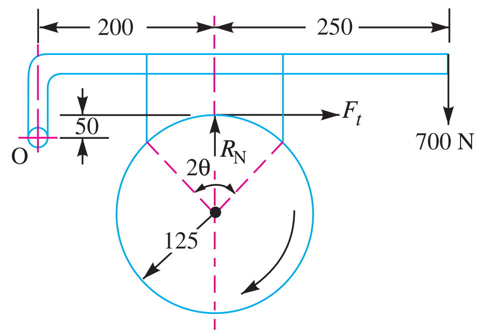
$$T_B = F_t \times r = \mu'.R_N.\ r$$

where μ' = Equivalent coefficient of friction = $\dfrac{4\mu \sin\theta}{2\,\theta + \sin 2\theta}$, and

μ = Actual coefficient of friction.

These brakes have more life and may provide a higher braking torque.





All dimensions in mm.

Fig. 25.4. Pivoted block or shoe brake. **Fig. 25.5**

Example 25.2. *A single block brake is shown in Fig. 25.5. The diameter of the drum is 250 mm and the angle of contact is 90°. If the operating force of 700 N is applied at the end of a lever and the coefficient of friction between the drum and the lining is 0.35, determine the torque that may be transmitted by the block brake.*

Solution. Given : d = 250 mm or r = 125 mm ; $2\theta = 90° = \pi / 2$ rad ; P = 700 N ; μ = 0.35

Since the angle of contact is greater than 60° , therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\,\theta + \sin 2\theta} = \frac{4 \times 0.35 \times \sin 45°}{\pi/2 + \sin 90°} = 0.385$$

Let R_N = Normal force pressing the block to the brake drum, and

F_t = Tangential braking force = $\mu'.\ R_N$

Taking moments above the fulcrum O, we have

$$700\,(250 + 200) + F_t \times 50 = R_N \times 200 = \frac{F_t}{\mu'} \times 200 = \frac{F_t}{0.385} \times 200 = 520\ F_t$$

or $520\ F_t - 50\ F_t = 700 \times 450$ or $F_t = 700 \times 450 / 470 = 670$ N

We know that torque transmitted by the block brake,

$$T_B = F_t \times r = 670 \times 125 = 83\ 750 \text{ N-mm} = 83.75 \text{ N-m}$$ **Ans.**

Example 25.3. *Fig. 25.6 shows a brake shoe applied to a drum by a lever AB which is pivoted at a fixed point A and rigidly fixed to the shoe. The radius of the drum is 160 mm. The coefficient of friction of the brake lining is 0.3. If the drum rotates clockwise, find the braking torque due to the horizontal force of 600 N applied at B.*

Solution. Given : r = 160 mm = 0.16 m ; μ = 0.3 ; P = 600 N

Since the angle subtended by the shoe at the centre of the drum is 40°, therefore we need not to calculate the equivalent coefficient of friction (μ').

Let R_N = Normal force pressing the shoe on the drum, and

F_t = Tangential braking force = $\mu.R_N$

Top

Contents

Taking moments about point A,

$$R_N \times 350 + F_t\,(200 - 160) = 600\,(400 + 350)$$

$$\frac{F_t}{0.3} \times 350 + 40\,F_t = 600 \times 750$$

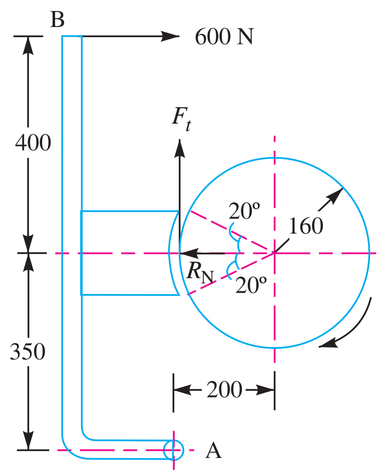
or $$207\,F_t = 450 \times 10^3$$

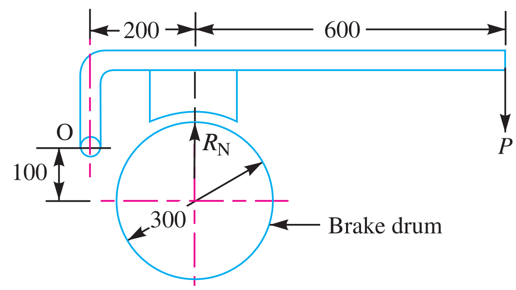
$$\therefore \quad F_t = 450 \times 10^3 / 1207 = 372.8 \text{ N}$$

We know that braking torque,

$$T_B = F_t \times r = 372.8 \times 0.16$$
$$= 59.65 \text{ N-m } \textbf{Ans.}$$

Brakes on a car wheel (inner side)





All dimensions in mm.

Fig. 25.6 **Fig. 25.7**

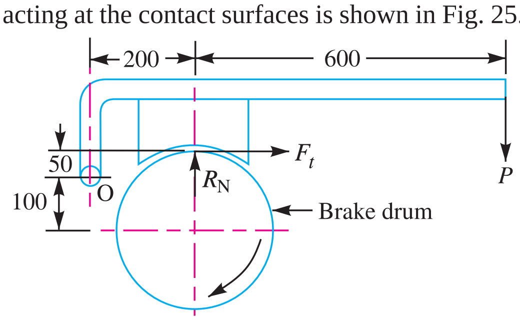
Example 25.4. *The block brake, as shown in Fig. 25.7, provides a braking torque of 360 N-m. The diameter of the brake drum is 300 mm. The coefficient of friction is 0.3. Find :*

1. *The force (P) to be applied at the end of the lever for the clockwise and counter clockwise rotation of the brake drum; and*
2. *The location of the pivot or fulcrum to make the brake self locking for the clockwise rotation of the brake drum.*

Solution. Given : T_B = 360 N-m = 360 × 10^3 N-mm ; d = 300 mm or r = 150 mm = 0.15 m ; μ = 0.3

1. ***Force (P) for the clockwise and counter clockwise rotation of the brake drum***

For the clockwise rotation of the brake drum, the frictional force or the tangential force (F_t) acting at the contact surfaces is shown in Fig. 25.8.



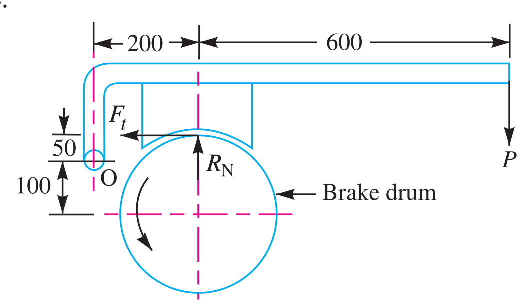


Fig. 25.8 **Fig. 25.9**

Top

Contents

We know that braking torque (T_B),

$$360 = F_t \times r = F_t \times 0.15 \quad \text{or} \quad F_t = 360 / 0.15 = 2400 \text{ N}$$

and normal force, $$R_N = F_t / \mu = 2400 / 0.3 = 8000 \text{ N}$$

Now taking moments about the fulcrum O, we have

$$P(600 + 200) + F_t \times 50 = R_N \times 200$$

$$P \times 800 + 2400 \times 50 = 8000 \times 200$$

$$P \times 800 = 8000 \times 200 - 2400 \times 50 = 1480 \times 10^3$$

$$\therefore \quad P = 1480 \times 10^3 / 800 = 1850 \text{ N} \quad \textbf{Ans.}$$

For the counter clockwise rotation of the drum, the frictional force or the tangential force (F_t) acting at the contact surfaces is shown in Fig. 25.9.

Taking moments about the fulcrum O, we have

$$P(600 + 200) = F_t \times 50 + R_N \times 200$$

$$P \times 800 = 2400 \times 50 + 8000 \times 200 = 1720 \times 10^3$$

$$\therefore \quad P = 1720 \times 10^3 / 800 = 2150 \text{ N} \quad \textbf{Ans.}$$

2. *Location of the pivot or fulcrum to make the brake self-locking*

The clockwise rotation of the brake drum is shown in Fig. 25.8. Let x be the distance of the pivot or fulcrum O from the line of action of the tangential force (F_t). Taking moments about the fulcrum O, we have

$$P(600 + 200) + F_t \times x - R_N \times 200 = 0$$

In order to make the brake self-locking, $F_t \times x$ must be equal to $R_N \times 200$ so that the force P is zero.

$$\therefore \quad F_t \times x = R_N \times 200$$

$$2400 \times x = 8000 \times 200 \quad \text{or} \quad x = 8000 \times 200 / 2400 = 667 \text{ mm} \quad \textbf{Ans.}$$

Example 25.5. *A rope drum of an elevator having 650 mm diameter is fitted with a brake drum of 1 m diameter. The brake drum is provided with four cast iron brake shoes each subtending an angle of 45°. The mass of the elevator when loaded is 2000 kg and moves with a speed of 2.5 m / s. The brake has a sufficient capacity to stop the elevator in 2.75 metres. Assuming the coefficient of friction between the brake drum and shoes as 0.2, find: 1. width of the shoe, if the allowable pressure on the brake shoe is limited to 0.3 N/mm²; and 2. heat generated in stopping the elevator.*

Solution. Given : $d_e = 650$ mm or $r_e = 325$ mm $= 0.325$ m ; $d = 1$ m or $r = 0.5$ m $= 500$ mm ; $n = 4$; $2\,\theta = 45°$ or $\theta = 22.5°$; $m = 2000$ kg ; $v = 2.5$ m / s ; $h = 2.75$ m ; $\mu = 0.2$; $p_b = 0.3$ N/mm^2

1. *Width of the shoe*

Let w = Width of the shoe in mm.

First of all, let us find out the acceleration of the rope (a). We know that

$$v^2 - u^2 = 2\,a.h \quad \text{or} \quad (2.5)^2 - 0 = 2a \times 2.75 = 5.5a$$

$$\therefore \quad a = (2.5)^2 / 5.5 = 1.136 \text{ m/s}^2$$

and accelerating force = Mass × Acceleration = $m \times a = 2000 \times 1.136 = 2272$ N

∴ Total load acting on the rope while moving,

$$W = \text{Load on the elevator in newtons} + \text{Accelerating force}$$
$$= 2000 \times 9.81 + 2272 = 21\,892 \text{ N}$$

We know that torque acting on the shaft,

$$T = W \times r_e = 21\,892 \times 0.325 = 7115 \text{ N-m}$$

Top

∴ Tangential force acting on the drum

$$= \frac{T}{r} = \frac{7115}{0.5} = 14\ 230\ \text{N}$$

The brake drum is provided with four cast iron shoes, therefore tangential force acting on each shoe,

$$F_t = 14\ 230 / 4 = 3557.5\ \text{N}$$

Since the angle of contact of each shoe is 45°, therefore we need not to calculate the equivalent coefficient of friction (μ′).

∴ Normal load on each shoe,

$$R_N = F_t / \mu = 3557.5 / 0.2 = 17\ 787.5\ \text{N}$$

We know that the projected bearing area of each shoe,

$$A_b = w\,(2r \sin \theta) = w\,(2 \times 500 \sin 22.5°) = 382.7\ w\ \text{mm}^2$$

We also know that bearing pressure on the shoe (p_b),

$$0.3 = \frac{R_N}{A_b} = \frac{17\ 787.5}{382.7\ w} = \frac{46.5}{w}$$

$$\therefore \quad w = 46.5 / 0.3 = 155\ \text{mm}$$ **Ans.**

2. Heat generated in stopping the elevator

We know that heat generated in stopping the elevator

= Total energy absorbed by the brake

$= \text{Kinetic energy} + \text{Potential energy} = \frac{1}{2}\, m.v^2 + m.g.h$

$= \frac{1}{2} \times 2000\,(2.5)^2 + 2000 \times 9.81 \times 2.75 = 60\ 205\ \text{N-m}$

= 60.205 kN-m = 60.205 kJ **Ans.**

25.8 Double Block or Shoe Brake

When a single block brake is applied to a rolling wheel, and additional load is thrown on the shaft bearings due to the normal force (R_N). This produces bending of the shaft. In order to overcome this drawback, a double block or shoe brake as shown in Fig. 25.10, is used. It consists of two brake blocks applied at the opposite ends of a diameter of the wheel which eliminate or reduces the unbalanced force on the shaft. The brake is set by a spring which pulls the upper ends of the brake arms together. When a force P is applied to the bell crank lever, the spring is compressed and the brake is released. This type of brake is often used on electric cranes and the force P is produced by an electromagnet or solenoid. When the current is switched off, there is no force on the bell crank lever and the brake is engaged automatically due to the spring force and thus there will be no downward movement of the load.

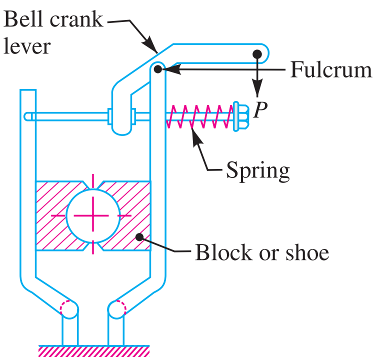


Fig. 25.10. Double block or shoe brake.

In a double block brake, the braking action is doubled by the use of two blocks and the two blocks may be operated practically by the same force which will operate one. In case of double block or shoe brake, the braking torque is given by

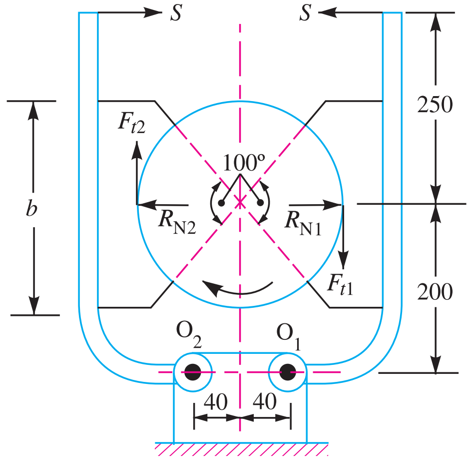
$$T_B = (F_{t1} + F_{t2})\, r$$

where F_{t1} and F_{t2} are the braking forces on the two blocks.

Example 25.6. *A double shoe brake, as shown in Fig. 25.11 is capable of absorbing a torque of 1400 N-m. The diameter of the brake drum is 350 mm and the angle of contact for each shoe is 100°. If the coefficient of friction between the brake drum and lining is 0.4; find : 1. the spring force*

Contents

necessary to set the brake; and 2. the width of the brake shoes, if the bearing pressure on the lining material is not to exceed 0.3 N/mm².



All dimensions in mm.

Fig. 25.11

Solution. Given : T_B = 1400 N-m = 1400 × 10³ N-mm ; d = 350 mm or r = 175 mm ; 2θ = 100° = 100 × π / 180 = 1.75 rad ; μ = 0.4 ; p_b = 0.3 N/mm²

1. ***Spring force necessary to set the brake***

Let S = Spring force necessary to set the brake,

R_{N1} and F_{t1} = Normal reaction and the braking force on the right hand side shoe, and

R_{N2} and F_{t2} = Corresponding values on the left hand side shoe.

Since the angle of contact is greater than 60°, therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.4 \times \sin 50°}{1.75 + \sin 100°} = 0.45$$

Taking moments about the fulcrum O_1, we have

$$S \times 450 = R_{N1} \times 200 + F_{t1}\ (175 - 40) = \frac{F_{t1}}{0.45} \times 200 + F_{t1} \times 135 = 579.4\ F_{t1}$$

...(Substituting $R_{N1} = F_{t1} / \mu'$)

$\therefore$ $F_{t1} = S \times 450 / 579.4 = 0.776\ S$

(A)

(B)

Train braking system : *(A) Flexible hose carries the brakepipe between car; (B) Brake hydraulic cylinder and the associated hardware.*

Top

Contents

(C) (D)

Train braking system : (C) Shoe of the train brake (D) Overview of train brake

Again taking moments about O_2, we have

$$S \times 450 + F_{t2}\,(175 - 40) = R_{N2} \times 200 = \frac{F_{t2}}{0.45} \times 200 = 444.4\,F_{t2}$$

...(Substituting $R_{N2} = F_{t2}/\mu'$)

$$444.4\,F_{t2} - 135\,F_{t2} = S \times 450 \quad \text{or} \quad 309.4\,F_{t2} = S \times 450$$

$$\therefore \quad F_{t2} = S \times 450 / 309.4 = 1.454\,S$$

We know that torque capacity of the brake (T_B),

$$1400 \times 10^3 = (F_{t1} + F_{t2})\,r = (0.776\,S + 1.454\,S)\,175 = 390.25\,S$$

$$\therefore \quad S = 1400 \times 10^3 / 390.25 = 3587 \text{ N} \quad \textbf{Ans.}$$

2. *Width of the brake shoes*

Let b = Width of the brake shoes in mm.

We know that projected bearing area for one shoe,

$$A_b = b\,(2r \sin\theta) = b\,(2 \times 175 \sin 50°) = 268\,b \text{ mm}^2$$

$\therefore$ Normal force on the right hand side of the shoe,

$$R_{N1} = \frac{F_{t1}}{\mu'} = \frac{0.776 \times S}{0.45} = \frac{0.776 \times 3587}{0.45} = 6186 \text{ N}$$

and normal force on the left hand side of the shoe,

$$R_{N2} = \frac{F_{t2}}{\mu'} = \frac{1.454 \times S}{0.45} = \frac{1.454 \times 3587}{0.45} = 11\,590 \text{ N}$$

We see that the maximum normal force is on the left hand side of the shoe. Therefore we shall design the shoe for the maximum normal force *i.e.* R_{N2}.

We know that the bearing pressure on the lining material (p_b),

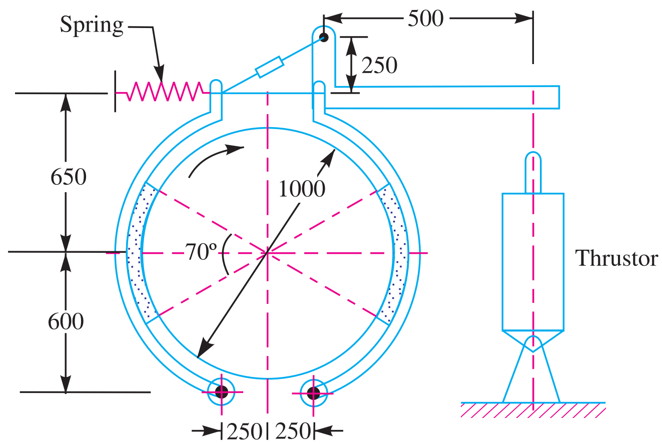
$$0.3 = \frac{R_{N2}}{A_b} = \frac{11\,590}{268\,b} = \frac{43.25}{b}$$

$$\therefore \quad b = 43.25 / 0.3 = 144.2 \text{ mm} \quad \textbf{Ans.}$$

Example 25.7. *A spring closed thrustor operated double shoe brake is to be designed for a maximum torque capacity of 3000 N-m. The brake drum diameter is not to exceed 1 metre and the shoes are to be lined with Ferrodo having a coefficient of friction 0.3. The other dimensions are as shown in Fig. 25.12.*

Top

Contents



All dimensions in mm.

Fig. 25.12

1. *Find the spring force necessary to set the brake.*
2. *If the permissible stress of the spring material is 500 MPa, determine the dimensions of the coil assuming spring index to be 6. The maximum spring force is to be 1.3 times the spring force required during braking. There are eight active coils. Specify the length of the spring in the closed position of the brake. Modulus of rigidity is 80 kN / mm^2.*
3. *Find the width of the brake shoes if the bearing pressure on the lining material is not to exceed 0.5 N/mm^2.*
4. *Calculate the force required to be exerted by the thrustor to release the brake.*

Solution. Given : T_B = 3000 N-m = 3 × 10^6 N-mm ; d = 1 m or r = 0.5 m = 500 mm ; μ = 0.3 ; 2θ = 70° = 70 × π / 180 = 1.22 rad

1. Spring force necessary to set the brake

Let S = Spring force necessary to set the brake,

R_{N1} and F_{t1} = Normal reaction and the braking force on the right hand side shoe,

and R_{N2} and F_{t2} = Corresponding values for the left hand side shoe.

Since the angle of contact is greater than 60°, therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.3 \times \sin 35°}{1.22 + \sin 70°} = 0.32$$

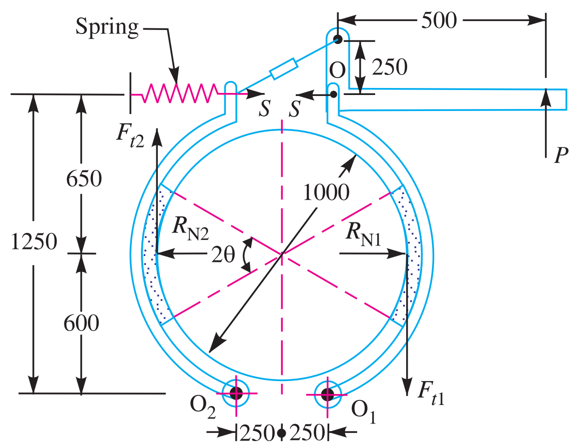
Taking moments about the fulcrum O_1 (Fig. 25.13), we have

$$S \times 1250 = R_{N1} \times 600 + F_{t1}\,(500 - 250)$$

$$= \frac{F_{t1}}{0.32} \times 600 + 250\, F_{t1} = 2125\, F_{t1} \qquad ...(\because\ R_{N1} = F_{t1} / \mu')$$

$$\therefore \quad F_{t1} = S \times 1250 / 2125 = 0.59\, S \text{ N}$$

Top



All dimensions in mm.

Fig. 25.13

Again taking moments about the fulcrum O_2, we have

$$S \times 1250 + F_{t2}\,(500 - 250) = R_{N2} \times 600 = \frac{F_{t2}}{0.32} \times 600 = 1875\ F_{t2} \qquad ...(\because R_{N2} = F_{t2}/\mu')$$

or $\quad 1875\ F_{t2} - 250\ F_{t2} = S \times 1250 \quad$ or $\quad 1625\ F_{t2} = S \times 1250$

$\therefore \quad F_{t2} = S \times 1250\ /\ 1625 = 0.77\ S$ N

We know that torque capacity of the brake (T_B),

$$3 \times 10^6 = (F_{t1} + F_{t2})\ r = (0.59\ S + 0.77\ S)\ 500 = 680\ S$$

$\therefore \quad S = 3 \times 10^6 / 680 = 4412$ N **Ans.**

2. *Dimensions of the spring coil*

Given : $\tau = 500$ MPa $= 500$ N/mm^2 ; $C = D/d = 6$; $n = 8$; $G = 80$ kN/mm$^2 = 80 \times 10^3$ N/mm^2

Let $\quad D$ = Mean diameter of the spring, and

$\quad d$ = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

Since the maximum spring force is 1.3 times the spring force required during braking, therefore maximum spring force,

$$W_S = 1.3\ S = 1.3 \times 4412 = 5736 \text{ N}$$

We know that the shear stress induced in the spring (τ),

$$500 = \frac{K \times 8 W_S . C}{\pi d^2} = \frac{1.2525 \times 8 \times 5736 \times 6}{\pi d^2} = \frac{109\ 754}{d^2}$$

$\therefore \quad d^2 = 109\ 754 / 500 = 219.5 \quad$ or $\quad d = 14.8$ say 15 mm **Ans.**

and $\quad D = C.d = 6 \times 15 = 90$ mm **Ans.**

We know that deflection of the spring,

$$\delta = \frac{8 W_S . C^3 . n}{G . d} = \frac{8 \times 5736 \times 6^3 \times 8}{80 \times 10^3 \times 15} = 66 \text{ mm}$$

Contents

The length of the spring in the closed position of the brake will be its free length. Assuming that the ends of the coil are squared and ground, therefore total number of coils,

$$n' = n + 2 = 8 + 2 = 10$$

∴ Free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta$$
$$= 10 \times 15 + 66 + 0.15 \times 66 = 226 \text{ mm}$$ **Ans.**

3. *Width of the brake shoes*

Let b = Width of the brake shoes in mm, and

p_b = Bearing pressure on the lining material of the shoes.

= 0.5 N/mm^2 ...(Given)

We know that projected bearing area for one shoe,

$$A_b = b\,(2r.\sin\theta) = b\,(2 \times 500 \sin 35°) = 574\,b \text{ mm}^2$$

We know that normal force on the right hand side of the shoe,

$$R_{N1} = \frac{F_{t1}}{\mu'} = \frac{0.59\ S}{0.32} = \frac{0.59 \times 4412}{0.32} = 8135 \text{ N}$$

and normal force on the left hand side of the shoe,

$$R_{N2} = \frac{F_{t2}}{\mu'} = \frac{0.77\ S}{0.32} = \frac{0.77 \times 4412}{0.32} = 10\ 616 \text{ N}$$

We see that the maximum normal force is on the left hand side of the shoe. Therefore we shall design the shoe for the maximum normal force *i.e.* R_{N2}.

We know that bearing pressure on the lining material (p_b),

$$0.5 = \frac{R_{N2}}{A_b} = \frac{10\ 616}{574\,b} = \frac{18.5}{b}$$

∴ $b = 18.5 / 0.5 = 37$ mm **Ans.**

4. *Force required to be exerted by the thrustor to release the brake*

Let P = Force required to be exerted by the thrustor to release the brake.

Taking moments about the fulcrum of the lever O, we have

$$P \times 500 + R_{N1} \times 650 = F_{t1}\,(500 - 250) + F_{t2}\,(500 + 250) + R_{N2} \times 650$$

$$P \times 500 + 8135 \times 650 = 0.59 \times 4412 + 250 + 0.77 \times 4412 \times 750 + 10\ 616 \times 650$$

...(Substituting $F_{t1} = 0.59\ S$ and $F_{t2} = 0.77\ S$)

$$P \times 500 + 5.288 \times 10^6 = 0.65 \times 10^6 + 2.55 \times 10^6 + 6.9 \times 10^6 = 10.1 \times 10^6$$

∴ $$P = \frac{10.1 \times 10^6 - 5.288 \times 10^6}{500} = 9624 \text{ N}$$ **Ans.**

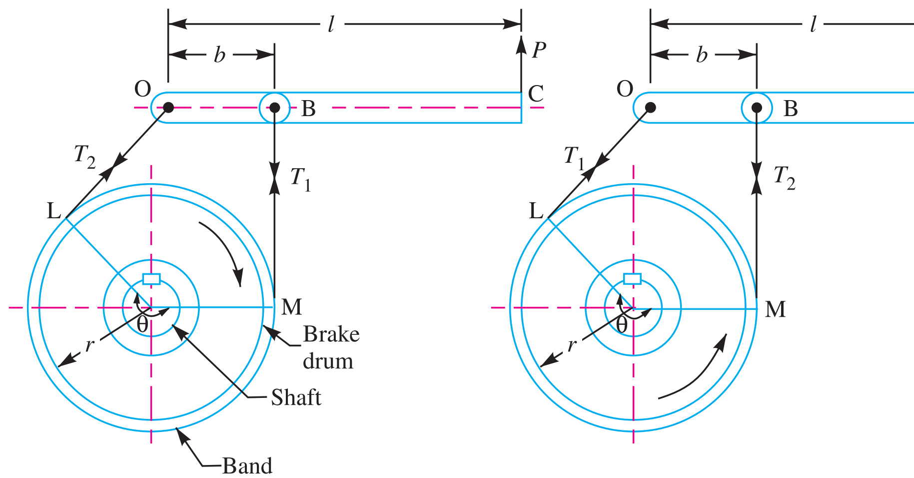
25.9 Simple Band Brake

A band brake consists of a flexible band of leather, one or more ropes, or a steel lined with friction material, which embraces a part of the circumference of the drum. A band brake, as shown in Fig. 25.14, is called a ***simple band brake*** in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance b from the fulcrum.

Top

Contents

When a force P is applied to the lever at C, the lever turns about the fulcrum pin O and tightens the band on the drum and hence the brakes are applied. The friction between the band and the drum provides the braking force. The force P on the lever at C may be determined as discussed below :



(*a*) Clockwise rotation of drum. (*b*) Anticlockwise rotation of drum.

Fig. 25.14. Simple band brake.

Let T_1 = Tension in the tight side of the band,

T_2 = Tension in the slack side of the band,

θ = Angle of lap (or embrace) of the band on the drum,

μ = Coefficient of friction between the band and the drum,

r = Radius of the drum,

t = Thickness of the band, and

r_e = Effective radius of the drum = $r + t / 2$.

We know that limiting ratio of the tensions is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad 2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

and braking force on the drum

$$= T_1 - T_2$$

∴ Braking torque on the drum,

$$T_B = (T_1 - T_2)\, r \qquad \text{...(Neglecting thickness of band)}$$

$$= (T_1 - T_2)\, r_e \qquad \text{...(Considering thickness of band)}$$

Now considering the equilibrium of the lever OBC. It may be noted that when the drum rotates in the clockwise direction as shown in Fig. 25.14 (*a*), the end of the band attached to the fulcrum O will be slack with tension T_2 and end of the band attached to B will be tight with tension T_1. On the other hand, when the drum rotates in the anticlockwise direction as shown in Fig. 25.14 (*b*), the tensions in the band will reverse, *i.e.* the end of the band attached to the fulcrum O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. Now taking moments about the fulcrum O, we have

$$P.l = T_1.b \qquad \text{...(for clockwise rotation of the drum)}$$

and

$$P.l = T_2.b \qquad \text{...(for anticlockwise rotation of the drum)}$$

Top

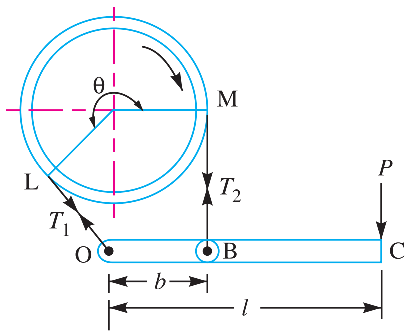
Contents

where l = Length of the lever from the fulcrum (OC), and

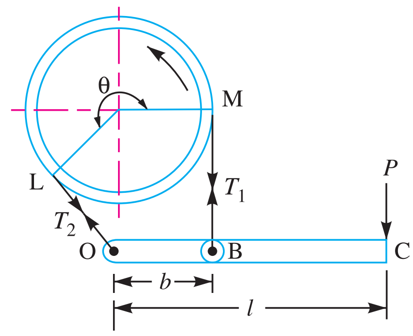
b = Perpendicular distance from O to the line of action of T_1 or T_2.

Notes: 1. When the brake band is attached to the lever, as shown in Fig. 25.14 (***a***) and (*b*), then the force (*P*) must act in the upward direction in order to tighten the band on the drum.

2. Sometimes the brake band is attached to the lever as shown in Fig. 25.15 (*a*) and (*b*), then the force (*P*) must act in the downward direction in order to tighten the band. In this case, for clockwise rotation of the drum, the end of the band attached to the fulcrum *O* will be tight with tension T_1 and band of the band attached to *B* will be slack with tension T_2. The tensions T_1 and T_2 will reverse for anticlockwise rotation of the drum.



(*a*) Clockwise rotation of drum.



(*b*) Anticlockwise rotation of drum.

Fig. 25.15. Simple band brake.

3. If the permissible tensile stress (σ_t) for the material of the band is known, then maximum tension in the band is given by

$$T_1 = \sigma_t \times w \times t$$

where w = Width of the band, and

t = Thickness of the band.

4. The width of band (*w*) should not exceed 150 mm for drum diameter (*d*) greater than 1 metre and 100 mm for drum diameter less than 1 metre. The band thickness (*t*) may also be obtained by using the empirical relation *i.e.* $t = 0.005\ d$

For brakes of hand operated winches, the steel bands of the following sizes are usually used :

Width of band (*w*) in mm	25 – 40	40 – 60	80	100	140 – 200
Thickness of band (*t*) in mm	3	3 – 4	4 – 6	4 – 7	6 – 10

Example 25.8. *A simple band brake operates on a drum of 600 mm in diameter that is running at 200 r.p.m. The coefficient of friction is 0.25. The brake band has a contact of 270°, one end is fastened to a fixed pin and the other end to the brake arm 125 mm from the fixed pin. The straight brake arm is 750 mm long and placed perpendicular to the diameter that bisects the angle of contact.*

(a) What is the pull necessary on the end of the brake arm to stop the wheel if 35 kW is being absorbed ? What is the direction for this minimum pull ?

(b) What width of steel band of 2.5 mm thick is required for this brake if the maximum tensile stress is not to exceed 50 MPa ?

Solution. Given : d = 600 mm or r = 300 mm ; N = 200 r.p.m. ; μ = 0.25 ; θ = 270° = 270 × π/180 = 4.713 rad ; Power = 35 kW = 35 × 10^3 W ; t = 2.5 mm ; σ_t = 50 MPa = 50 N/mm^2

(a) Pull necessary on the end of the brake arm to stop the wheel

Let P = Pull necessary on the end of the brake arm to stop the wheel.

Top

Band brake

Bands of a brake shown separately

The simple band brake is shown in Fig. 25.16. Since one end of the band is attached to the fixed pin O, therefore the pull P on the end of the brake arm will act upward and when the wheel rotates anticlockwise, the end of the band attached to O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. First of all, let us find the tensions T_1 and T_2. We know that

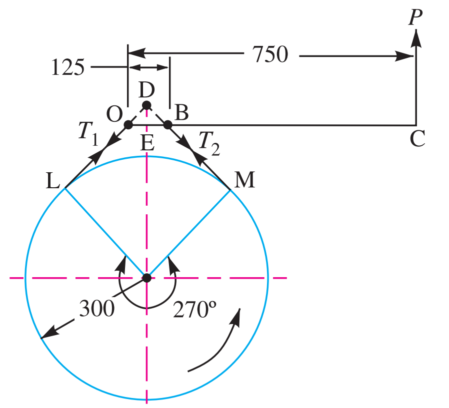
$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 4.713 = 1.178$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = 1.178 / 2.3 = 0.5123$$

or

$$\frac{T_1}{T_2} = 3.25 \quad \ldots(i)$$

...(Taking antilog of 0.5123)



All dimensions in mm.

Fig. 25.16

Let T_B = Braking torque.

We know that power absorbed,

$$35 \times 10^3 = \frac{2\pi N.T_B}{60} = \frac{2\pi \times 200 \times T_B}{60} = 21\, T_B$$

$$\therefore \quad T_B = 35 \times 10^3 / 21 = 1667 \text{ N-m} = 1667 \times 10^3 \text{ N-mm}$$

We also know that braking torque (T_B),

$$1667 \times 10^3 = (T_1 - T_2)\, r = (T_1 - T_2)\, 300$$

$$\therefore \quad T_1 - T_2 = 1667 \times 10^3 / 300 = 5557 \text{ N} \quad \ldots(ii)$$

From equations **(*i*)** and **(*ii*)**, we find that

$$T_1 = 8027 \text{ N} ; \text{ and } T_2 = 2470 \text{ N}$$

Now taking moments about O, we have

$$P \times 750 = T_2 \times {}^*OD = T_2 \times 62.5\sqrt{2} = 2470 \times 88.4 = 218\,348$$

$$\therefore \quad P = 218\,348 / 750 = 291 \text{ N}$$ **Ans.**

* OD = Perpendicular distance from O to the line of action of tension T_2.

$OE = EB = OB / 2 = 125 / 2 = 62.5$ mm, and $\angle DOE = 45°$

$\therefore \quad OD = OE \sec 45° = 62.5\sqrt{2}$ mm

Contents

(*b*) *Width of steel band*

Let w = Width of steel band in mm.

We know that maximum tension in the band (T_1),

$$8027 = \sigma_t \times w \times t = 50 \times w \times 2.5 = 125\, w$$

$$\therefore \quad w = 8027/125 = 64.2 \text{ mm}$$ **Ans.**

Example 25.9. *A band brake acts on the $\frac{3}{4}$th of circumference of a drum of 450 mm diameter which is keyed to the shaft. The band brake provides a braking torque of 225 N-m. One end of the band is attached to a fulcrum pin of the lever and the other end to a pin 100 mm from the fulcrum. If the operating force applied at 500 mm from the fulcrum and the coefficient of friction is 0.25, find the operating force when the drum rotates in the anticlockwise direction.*

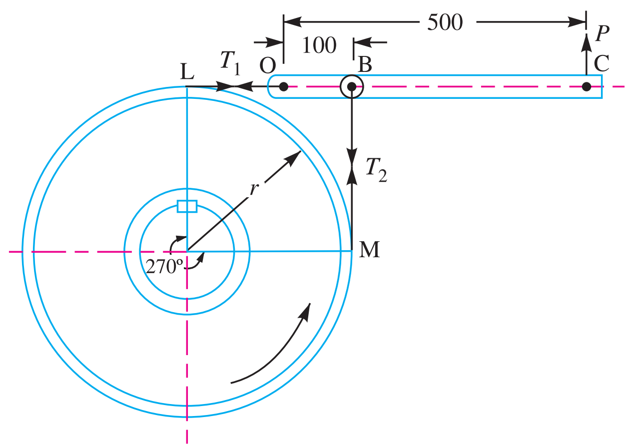
If the brake lever and pins are to be made of mild steel having permissible stresses for tension and crushing as 70 MPa and for shear 56 MPa, design the shaft, key, lever and pins. The bearing pressure between the pin and the lever may be taken as 8 N/mm^2.

Solution. Given : d = 450 mm or r = 225 mm ; T_B = 225 N-m = 225 × 10^3 N-mm ; OB = 100 mm ; l = 500 mm ; μ = 0.25 ; $\sigma_t = \sigma_c$ = 70 MPa = 70 N/mm^2 ; τ = 56 MPa = 56 N/mm^2 ; p_b = 8 N/mm^2

Operating force

Let P = Operating force.

The band brake is shown in Fig. 25.17. Since one end of the band is attached to the fulcrum at O, therefore the operating force P will act upward and when the drum rotates anticlockwise, the end of the band attached to O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. First of all, let us find the tensions T_1 and T_2.



All dimensions in mm.

Fig. 25.17

We know that angle of wrap,

$$\theta = \frac{3}{4}\text{th of circumference} = \frac{3}{4} \times 360° = 270°$$

$$= 270 \times \frac{\pi}{180} = 4.713 \text{ rad}$$

Top

and $$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 4.713 = 1.178$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{1.178}{2.3} = 0.5123 \quad \text{or} \quad \frac{T_1}{T_2} = 3.25 \qquad ...(i)$$

...(Taking antilog of 0.5123)

Drums for band brakes.

We know that braking torque (T_B),

$$225 \times 10^3 = (T_1 - T_2)\, r = (T_1 - T_2)\, 225$$

$$\therefore \quad T_1 - T_2 = 225 \times 10^3 / 225 = 1000 \text{ N} \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$T_1 = 1444 \text{ N and } T_2 = 444 \text{ N}$$

Taking moments about the fulcrum O, we have

$$P \times 500 = T_2 \times 100 = 444 \times 100 = 44\,400$$

$$\therefore \quad P = 44\,400 / 500 = 88.8 \text{ N} \quad \textbf{Ans.}$$

Design of shaft

Let d_s = Diameter of the shaft in mm.

Since the shaft has to transmit torque equal to the braking torque (T_B), therefore

$$225 \times 10^3 = \frac{\pi}{16} \times \tau\, (d_s)^3 = \frac{\pi}{16} \times 56\, (d_s)^3 = 11\, (d_s)^3$$

$$\therefore \quad (d_s)^3 = 225 \times 10^3 / 11 = 20.45 \times 10^3 \text{ or } d_s = 27.3 \text{ say } 30 \text{ mm} \quad \textbf{Ans.}$$

Design of key

The standard dimensions of the key for a 30 mm diameter shaft are as follows :

Width of key, w = 10 mm **Ans.**

Thickness of key, t = 8 mm **Ans.**

Let l = Length of key.

Considering the key in shearing, we have braking torque (T_B),

$$225 \times 10^3 = l \times w \times \tau \times \frac{d_s}{2} = l \times 10 \times 56 \times \frac{30}{2} = 8400\, l$$

$$\therefore \quad l = 225 \times 10^3 / 8400 = 27 \text{ mm}$$

Now considering the key in crushing, we have braking torque (T_B),

$$225 \times 10^3 = l \times \frac{t}{2} \times \sigma_c \times \frac{d_s}{2} = l \times \frac{8}{2} \times 70 \times \frac{30}{2} = 4200\, l$$

$$\therefore \quad l = 225 \times 10^3 / 4200 = 54 \text{ mm}$$

Taking larger of two values, we have l = 54 mm **Ans.**

Design of lever

Let t_1 = Thickness of the lever in mm, and

B = Width of the lever in mm.

The lever is considered as a cantilever supported at the fulcrum O. The effect of T_2 on the lever for determining the bending moment on the lever is neglected. This error is on the safer side.

∴ Maximum bending moment at O due to the force P,

$$M = P \times l = 88.8 \times 500 = 44\,400 \text{ N-m}$$

Section modulus,

$$Z = \frac{1}{6}\, t_1.B^2 = \frac{1}{6}\, t_1\, (2t_1)^2 = 0.67\, (t_1)^3 \text{ mm}^3 \qquad ...(\text{Assuming } B = 2t_1)$$

Contents

We know that the bending stress (σ_t),

$$70 = \frac{M}{Z} = \frac{44\ 400}{0.67\ (t_1)^3} = \frac{66\ 300}{(t_1)^3}$$

$\therefore$ $(t_1)^3 = 66\ 300 / 70 = 947$ or $t_1 = 9.82$ say 10 mm **Ans.**

and $B = 2\ t_1 = 2 \times 10 = 20$ mm **Ans.**

Design of pins

Let d_1 = Diameter of the pins at O and B, and

l_1 = Length of the pins at O and $B = 1.25\ d_1$ ...(Assume)

The pins at O and B are designed for the maximum tension in the band (*i.e.* T_1 = 1444 N),

Considering bearing of the pins at O and B, we have maximum tension (T_1),

$$1444 = d_1.l_1.p_b = d_1 \times 1.25\ d_1 \times 8 = 10\ (d_1)^2$$

$\therefore$ $(d_1)^2 = 1444 / 10 = 144.4$ or $d_1 = 12$ mm **Ans.**

and $l_1 = 1.25\ d_1 = 1.25 \times 12 = 15$ mm **Ans.**

Let us now check the pin for induced shearing stress. Since the pin is in double shear, therefore maximum tension (T_1),

$$1444 = 2 \times \frac{\pi}{4}\ (d_1)^2\ \tau = 2 \times \frac{\pi}{4}\ (12)^2\ \tau = 226\ \tau$$

$\therefore$ $\tau = 1444 / 226 = 6.4$ N/mm^2 = 6.4 MPa

This induced stress is quite within permissible limits.

The pin may be checked for induced bending stress. We know that maximum bending moment,

$$M = \frac{5}{24} \times W.l_1 = \frac{5}{24} \times 1444 \times 15 = 4513 \text{ N-mm}$$

... (Here $W = T_1$ = 1444 N)

and section modulus, $Z = \dfrac{\pi}{32}\ (d_1)^3 = \dfrac{\pi}{32}\ (12)^3 = 170\ \text{mm}^3$

$\therefore$ Bending stress induced

$$= \frac{M}{Z} = \frac{4513}{170} = 26.5\ \text{N-mm}^2 = 26.5\ \text{MPa}$$

This induced bending stress is within safe limits of 70 MPa.

The lever has an eye hole for the pin and connectors at band have forked end.

Thickness of each eye,

$$t_2 = \frac{l_1}{2} = \frac{15}{2} = 7.5 \text{ mm}$$

Outer diameter of the eye,

$$D = 2d_1 = 2 \times 12 = 24 \text{ mm}$$

A clearance of 1.5 mm is provided on either side of the lever in the fork.

A brass bush of 3 mm thickness may be provided in the eye of the lever.

$\therefore$ Diameter of hole in the lever

$$= d_1 + 2 \times 3 = 12 + 6 = 18 \text{ mm}$$

Top

Contents

The boss is made at pin joints whose outer diameter is taken equal to twice the diameter of the pin and length equal to length of the pin.

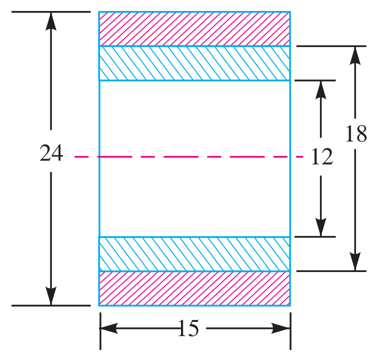
The inner diameter of the boss is equal to diameter of hole in the lever.

∴ Outer diameter of boss

$$= 2\, d_1 = 2 \times 12 = 24 \text{ mm}$$

and length of boss $= l_1 = 15$ mm

Let us now check the bending stress induced in the lever at the fulcrum. The section of the lever at the fulcrum is shown in Fig. 25.18.



All dimensions in mm.

Fig. 25.18

We know that maximum bending moment at the fulcrum,

$$M = P.l = 88.8 \times 500$$

$$= 44\,400 \text{ N-mm}$$

and section modulus,

$$Z = \frac{\frac{1}{12} \times 15\left[(24)^3 - (18)^3\right]}{24/2}$$

$$= 833 \text{ mm}^3$$

∴ Bending stress induced

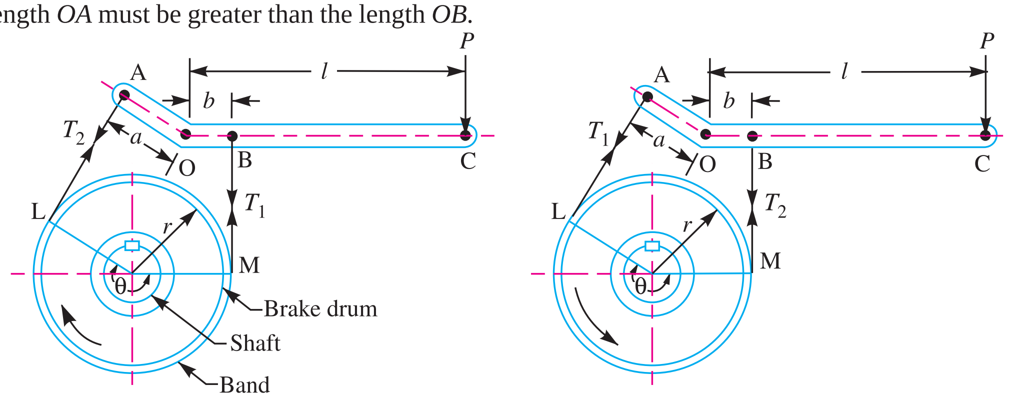
$$= \frac{M}{Z} = \frac{44\,400}{833} = 53.3 \text{ N/mm}^2$$

$$= 53.3 \text{ MPa}$$

This induced stress is within safe limits of 70 MPa.

25.10 Differential Band Brake

In a differential band brake, as shown in Fig. 25.19, the ends of the band are joined at *A* and *B* to a lever *AOC* pivoted on a fixed pin or fulcrum *O*. It may be noted that for the band to tighten, the length *OA* must be greater than the length *OB*.



(*a*) Clockwise rotation of the drum. (*b*) Anticlockwise rotation of the drum.

Fig. 25.19. Differenctial band brake.

The braking torque on the drum may be obtained in the similar way as discussed in simple band brake. Now considering the equilibrium of the lever *AOC*. It may be noted that when the drum rotates in the clockwise direction, as shown in Fig. 25.19 (*a*), the end of the band attached to *A* will be slack with tension T_2 and end of the band attached to *B* will be tight with tension T_1. On the other hand, when the drum rotates in the anticlockwise direction, as shown in Fig. 25.19 (*b*), the end of the band attached to *A* will be tight with tension T_1 and end of the band attached to *B* will be slack with tension T_2. Now taking moments about the fulcrum *O*, we have

Top

$$P.l + T_1.b = T_2.a \quad \text{...(for clockwise rotation of the drum)}$$

or $$P.l = T_2.a - T_1.b \quad \text{...}(i)$$

and $$P.l + T_2.b = T_1.a \quad \text{...(for anticlockwise rotation of the drum)}$$

or $$P.l = T_1.a - T_2.b \quad \text{...}(ii)$$

We have discussed in block brakes (Art. 25.6), that when the frictional force helps to apply the brake, it is said to be self energizing brake. In case of differential band brake, we see from equations (*i*) and (*ii*) that the moment $T_1.b$ and $T_2.b$ helps in applying the brake (because it adds to the moment $P.l$) for the clockwise and anticlockwise rotation of the drum respectively.

We have also discussed that when the force P is negative or zero, then brake is self locking. Thus for differential band brake and for clockwise rotation of the drum, the condition for self- locking is

$$T_2.a \le T_1.b \quad \text{or} \quad T_2/T_1 \le b / a$$

and for anticlockwise rotation of the drum, the condition for self-locking is

$$T_1.a \le T_2.b \quad \text{or} \quad T_1/T_2 \le b / a$$

Notes: 1. The condition for self-locking may also be written as follows. For clockwise rotation of the drum,

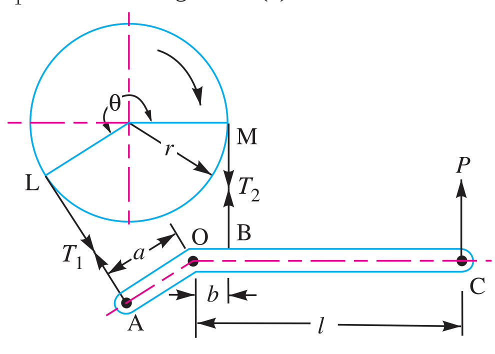
$$T_1.b \ge T_2.a \quad \text{or} \quad T_1/T_2 \ge a / b$$

and for anticlockwise rotation of the drum,

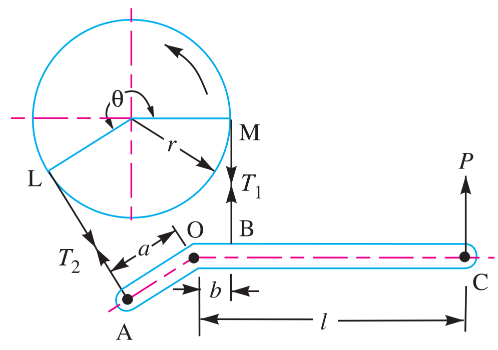
$$T_2.b \ge T_1.a \quad \text{or} \quad T_2/T_1 \ge a/b$$

2. When in Fig. 25.19 (*a*) and (*b*), the length OB is greater than OA, then the force P must act in the upward direction in order to apply the brake. The tensions in the band, *i.e.* T_1 and T_2 will remain unchanged.

3. Sometimes, the band brake is attached to the lever as shown in Fig. 25.20 (*a*) and (*b*). In such cases, when OA is greater than OB, the force (P) must act upwards. When the drum rotates in the clockwise direction, the end of the band attached to A will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2, as shown in Fig. 25.20 (*a*). When the drum rotates in the anticlockwise direction, the end of the band attached to A will be slack with tension T_2 and the end of the band attached to B will be tight with tension T_1, as shown in Fig. 25.20 (*b*).



(*a*) Clockwise rotation of the drum.



(*b*) Anticlockwise rotation of the drum.

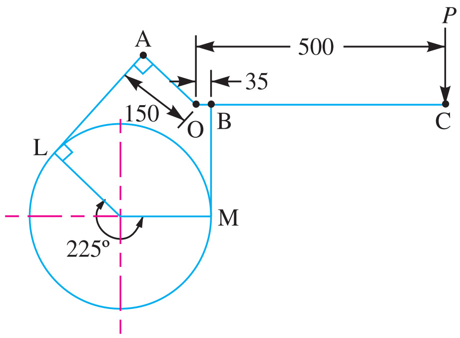
Fig. 25.20. Differential band brake.

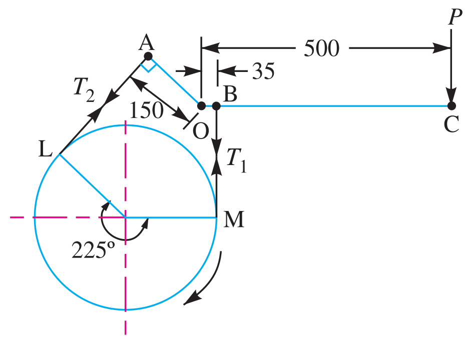
4. When in Fig. 25.20 (*a*) and (*b*), the length OB is greater than OA, then the force (P) must act downward in order to apply the brake. The position of tensions T_1 and T_2 will remain unchanged.

Example 25.10. *A differential band brake, as shown in Fig. 25.21, has an angle of contact of 225°. The band has a compressed woven lining and bears against a cast iron drum of 350 mm diameter. The brake is to sustain a torque of 350 N-m and the coefficient of friction between the band and the drum is 0.3. Find : 1. the necessary force (P) for the clockwise and anticlockwise rotation of the drum; and 2. The value of 'OA' for the brake to be self locking, when the drum rotates clockwise.*

Contents

Solution. Given : θ = 225° = 225 × π / 180 = 3.93 rad ; d = 350 mm or r = 175 mm ; T = 350 N-m = 350 × 10^3 N-mm ; μ = 0.3





All dimensions in mm.

Fig. 25.21 **Fig. 25.22**

1. ***Necessary force (P) for the clockwise and anticlockwise rotation of the drum***

When the drum rotates in the clockwise direction, the end of the band attached to A will be slack with tension T_2 and the end of the band attached to B will be tight with tension T_1, as shown in Fig. 25.22. First of all, let us find the values of tensions T_1 and T_2.

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 3.93 = 1.179$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{1.179}{2.3} = 0.5126$$

or $$\frac{T_1}{T_2} = 3.256 \qquad \text{...(Taking antilog of 0.5126)} \qquad ...(i)$$

and braking torque (T_B),

$$350 \times 10^3 = (T_1 - T_2)\, r = (T_1 - T_2)\, 175$$

$$\therefore \quad T_1 - T_2 = 350 \times 10^3 / 175 = 2000 \text{ N} \qquad ...(ii)$$

Another picture of car brake shoes

Top

Contents

From equations **(*i*)** and **(*ii*)**, we find that

$$T_1 = 2886.5 \text{ N} \; ; \text{ and } T_2 = 886.5 \text{ N}$$

Now taking moments about the fulcrum *O*, we have

$$P \times 500 = T_2 \times 150 - T_1 \times 35$$

or $$P \times 500 = 886.5 \times 150 - 2886.5 \times 35 = 31\,947.5$$

$\therefore$ $$P = 31\,947.5 / 500 = 64 \text{ N} \quad \textbf{Ans.}$$

When the drum rotates in the anticlockwise direction, the end of the band attached to *A* will be tight with tension T_1 and end of the band attached to *B* will be slack with tension T_2, as shown in Fig. 25.23. Taking moments about the fulcrum *O*, we have

$$P \times 500 = T_1 \times 150 - T_2 \times 35$$

or $$P \times 500 = 2886.5 \times 150 - 886.5 \times 35$$

$$= 401\,947.5$$

$\therefore$ $$P = 401\,947.5/500 = 804 \text{ N} \quad \textbf{Ans.}$$

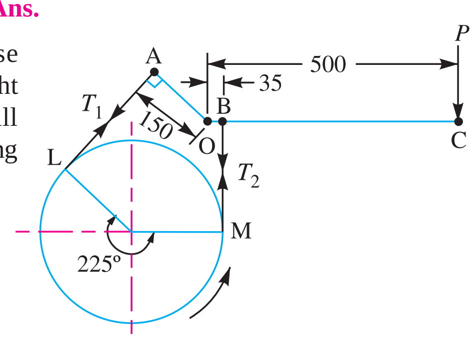


Fig. 25.23

2. *Valve of 'OA' for the brake to be self locking, when the drum rotates clockwise*

The clockwise rotation of the drum is shown in Fig. 25.22.

For clockwise rotation of the drum, we know that

$$P \times 500 + T_1 \times OB = T_2 \times OA$$

or $$P \times 500 = T_2 \times OA - T_1 \times OB$$

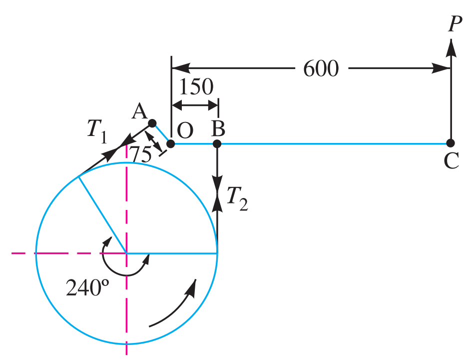
For the brake to be self-locking, *P* must be equal to zero or

$$T_2 \times OA = T_1 \times OB$$

or $$OA = \frac{T_1 \times OB}{T_2} = \frac{2886.5 \times 35}{886.5} = 114 \text{ mm} \quad \textbf{Ans.}$$

Example 25.11. *A differential band brake, as shown in Fig. 25.24, has a drum diameter of 600 mm and the angle of contact is 240°. The brake band is 5 mm thick and 100 mm wide. The coefficient of friction between the band and the drum is 0.3. If the band is subjected to a stress of 50 MPa, find :*

1. *The least force required at the end of a 600 mm lever, and*
2. *The torque applied to the brake drum shaft.*



All dimensions in mm.

Fig. 25.24

Solution. Given : d = 600 mm or r = 300 mm = 0.3 m ; θ = 240° = 240 × π / 180 = 4.2 rad ; t = 5 mm ; w = 100 mm ; μ = 0.3; σ_t = 50 MPa = 50 N/mm^2

1. *Least force required at the end of a lever*

Let P = Least force required at the end of the lever.

Top

Contents

Since the length *OB* is greater than *OA*, therefore the force at the end of the lever (*P*) must act in the upward direction. When the drum rotates anticlockwise, the end of the band attached to *A* will be tight with tension T_1 and the end of the band attached to *B* will be slack with tension T_2. First of all, let us find the values of tensions T_1 and T_2. We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 4.2 = 1.26$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{1.26}{2.3} = 0.5478 \quad \text{or} \quad \frac{T_1}{T_2} = 3.53 \quad \text{... (Taking antilog of 0.5478)} \quad ...(i)$$

We know that maximum tension in the band,

$$T_1 = \text{Stress} \times \text{Area of band} = \sigma_t \times t \times w$$

$$= 50 \times 5 \times 100 = 25\ 000 \text{ N}$$

and $$T_2 = T_1 / 3.53 = 25\ 000 / 3.53 = 7082 \text{ N} \quad \text{...[From equation } (i)]$$

Now taking moments about the fulcrum *O*, we have

$$P \times 600 + T_1 \times 75 = T_2 \times 150$$

$$\therefore \quad P = \frac{T_2 \times 150 - T_1 \times 75}{600} = \frac{7082 \times 150 - 25\ 000 \times 75}{600} = -1355 \text{ N}$$

$$= 1355 \text{ N (in magnitude)}$$ **Ans.**

Since *P* is negative, therefore the brake is self-locking.

2. *Torque applied to the brake drum shaft*

We know that torque applied to the brake drum shaft,

$$T_B = (T_1 - T_2)\, r = (25\ 000 - 7082)\ 0.3 = 5375 \text{ N-m}$$ **Ans.**

Example 25.12. *A differential band brake has a force of 220 N applied at the end of a lever as shown in Fig. 25.25. The coefficient of friction between the band and the drum is 0.4. The angle of lap is 180°. Find :*

1. The maximum and minimum force in the band, when a clockwise torque of 450 N-m is applied to the drum; and

2. The maximum torque that the brake may sustain for counter clockwise rotation of the drum.

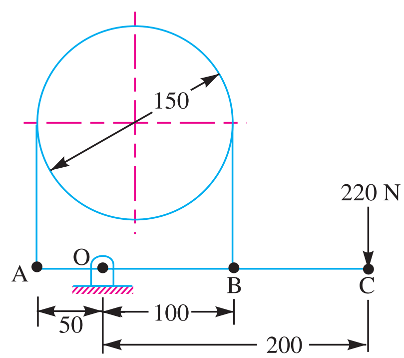


Fig. 25.25

Solution. Given : $P = 220$ N ; $\mu = 0.4$; $\theta = 180° = \pi$ rad ; $d = 150$ mm or $r = 75$ mm $= 0.075$ m

1. *Maximum and minimum force in the band*

Let T_1 = Maximum force in the band,

T_2 = Minimum force in the band,

and T_B = Torque applied to the drum = 450 N-m ...(Given)

In a differential band brake, when *OB* is greater than *OA* and the clockwise torque (T_B) is applied to the drum, then the maximum force (T_1) will be in the band attached to *A* and the minimum force (T_2) will be in the band attached to *B*, as shown in Fig. 25.26.

We know that braking torque (T_B),

$$450 = (T_1 - T_2)\, r = (T_1 - T_2)\ 0.075$$

$$\therefore \quad T_1 - T_2 = 450 / 0.075 = 6000 \text{ N}$$

or $$T_1 = (T_2 + 6000) \text{ N} \quad ...(i)$$

Top

Now taking moments about the pivot O, we have

$$220 \times 200 + T_1 \times 50 = T_2 \times 100$$

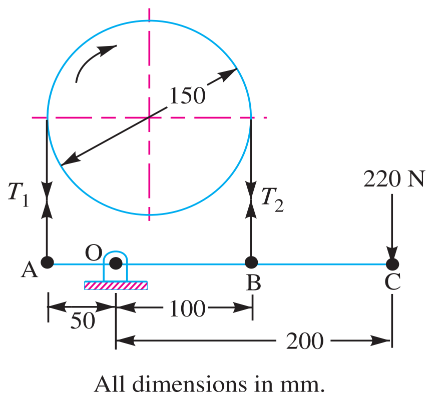
$$44\,000 + (T_2 + 6000)\,50 = T_2 \times 100$$

$$44\,000 + 50\,T_2 + 300\,000 = T_2 \times 100$$

or $\quad T_2 = 6880$ N **Ans.**

and $\quad T_1 = T_2 + 6000$

$= 6880 + 6000 = 12\,880$ N **Ans.** $\quad$ ...[From equation (*i*)]



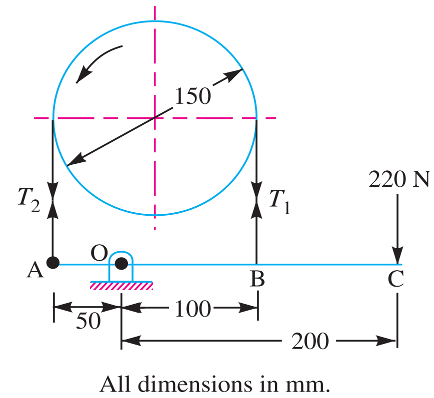


Fig. 25.26 $\qquad$ **Fig. 25.27**

2. Maximum torque that the brake may sustain for counter clockwise rotation of the drum.

When the drum rotates in the counter clockwise direction, the maximum force (T_1) will be in the band attached to B and the minimum force (T_2) will be in the band attached to A, as shown in Fig. 25.27. We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.4 \times \pi = 1.257$$

$$\log \left(\frac{T_1}{T_2}\right) = \frac{1.257}{2.3} = 0.5465$$

$\therefore \quad \dfrac{T_1}{T_2} = 3.52 \quad$ ...(Taking antilog of 0.5465) $\quad$ ...(*i*)

Now taking moments about the pivot O, we have

$220 \times 200 + T_2 \times 50 = T_1 \times 100 = 3.52\,T_2 \times 100 = 352\,T_2 \quad$ ...[From equation (*i*)]

or $\quad 44\,000 = 352\,T_2 - 50\,T_2 = 302\,T_2$

$\therefore \quad T_2 = 44\,000 / 302 = 146$ N

and $\quad T_1 = 3.52\,T_2 = 3.52 \times 146 = 514$ N

We know that the maximum torque that the brake may sustain,

$T_B = (T_1 - T_2)\,r = (514 - 146)\,0.075 = 27.6$ N-m **Ans.**

Example 25.13. *A differential band brake is operated by a lever of length 500 mm. The brake drum has a diameter of 500 mm and the maximum torque on the drum is 1000 N-m. The band brake embraces 2/3rd of the circumference. One end of the band is attached to a pin 100 mm from the fulcrum and the other end to another pin 80 mm from the fulcrum and on the other side of it when the operating force is also acting. If the band brake is lined with asbestos fabric having a coefficient of friction 0.3, find the operating force required.*

Design the steel band, shaft, key, lever and fulcrum pin. The permissible stresses may be taken as 70 MPa in tension, 50 MPa in shear and 20 MPa in bearing. The bearing pressure for the brake lining should not exceed 0.2 N/mm^2.

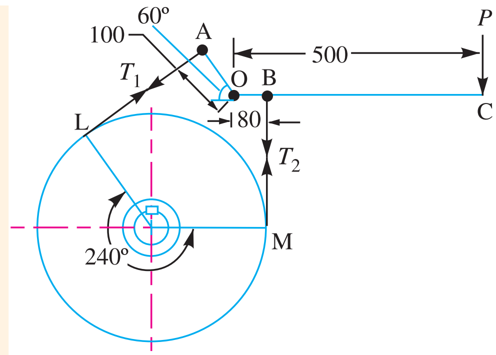


Fig. 25.28

Solution. Given : l = 500 mm ; d = 500 mm or r = 250 mm ; T_B = 1000 N-m = 1 × 10^6 N-mm ; OA = 100 mm ; OB = 80 mm ; μ = 0.3 ; σ_t = 70 MPa = 70 N/mm^2 ; τ = 50 MPa = 50 N/mm^2 ; σ_b = 20 MPa = 20 N/mm^2 ; p_b = 0.2 N/mm^2

Operating force

Let P = Operating force

The differential band brake is shown in Fig. 25.28. Since $OA > OB$, therefore the operating force (P) will act downward. When the drum rotates anticlockwise, the end of the band attached to A will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. First of all, let us find the values of tensions T_1 and T_2.

We know that angle of wrap,

$$\theta = \frac{2}{3}\text{rd of circumference}$$

$$= \frac{2}{3} \times 360° = 240° = 240 \times \frac{\pi}{180} = 4.19 \text{ rad}$$

and
$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 4.19 = 1.257$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{1.257}{2.3} = 0.5465 \quad \text{or} \quad \frac{T_1}{T_2} = 3.52 \quad \text{...(Taking antilog of 0.5465)} \quad \text{...(i)}$$

We know that the braking torque (T_B),

$$1 \times 10^6 = (T_1 - T_2)\, r = (T_1 - T_2)\, 250$$

$$\therefore \quad T_1 - T_2 = 1 \times 10^6 / 250 = 4000 \text{ N} \quad \text{...(ii)}$$

From equations (*i*) and (*ii*), we have

$$T_1 = 5587 \text{ N}, \text{ and } T_2 = 1587 \text{ N}$$

Now taking moments about the fulcrum O, we have

$$P \times 500 = T_1 \times 100 - T_2 \times 80$$

$$= 5587 \times 100 - 1587 \times 80$$

$$= 431\,740$$

$$\therefore \quad P = 431\,740 / 500 = 863.5 \text{ N} \quad \textbf{Ans.}$$

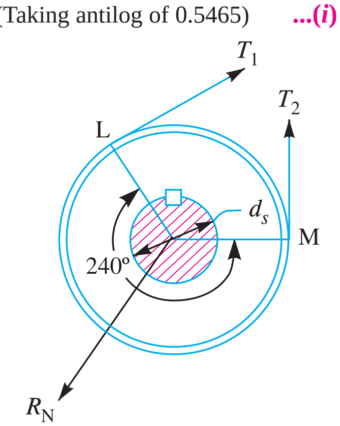


Fig. 25.29

Design for steel band

Let t = Thickness of band in mm, and

b = Width of band in mm.

We know that length of contact of band, as shown in Fig. 25.29,

$$= \pi d \times \frac{240}{360} = \pi \times 500 \times \frac{240}{360} \text{ mm} = 1047 \text{ mm}$$

∴ Area of contact of the band,

$$A_b = \text{Length} \times \text{Width of band} = 1047\, b \text{ mm}^2$$

We know that normal force acting on the band,

$$R_N = \frac{T_1 - T_2}{\mu} = \frac{5587 - 1587}{0.3} = 13\,333 \text{ N}$$

We also know that normal force on the band (R_N),

$$13\,333 = p_b \times A_b = 0.2 \times 1047\, b = 209.4\, b$$

∴ $b = 13\,333/209.4 = 63.7$ say 64 mm **Ans.**

and cross-sectional area of the band,

$$A = b \times t = 64\, t \text{ mm}^2$$

∴ Tensile strength of the band

$$= A \times \sigma_t = 64\, t \times 70 = 4480\, t \text{ N}$$

Since the band has to withstand a maximum tension (T_1) equal to 5587 N, therefore

$4480\, t = 5587$ or $t = 5587 / 4480 = 1.25$ mm **Ans.**

Design of shaft

Let d_s = Diameter of the shaft in mm.

Since the shaft has to transmit torque equal to the braking torque (T_B), therefore

$$1 \times 10^6 = \frac{\pi}{16} \times \tau\, (d_s)^3 = \frac{\pi}{16} \times 50\, (d_s)^3 = 9.82\, (d_s)^3$$

∴ $(d_s)^3 = 1 \times 10^6 / 9.82 = 101\,833$ or $d_s = 46.7$ say 50 mm **Ans.**

Design of key

The standard dimensions of the key for a 50 mm diameter shaft are as follows :

Width of key, w = 16 mm

Thickness of key, t_1 = 10 mm

Let l = Length of the key.

Considering shearing of the key, we have braking torque (T_B),

$$1 \times 10^6 = l \times w \times \tau \times \frac{d_s}{2} = l \times 16 \times 50 \times \frac{50}{2} = 20 \times 10^3\, l$$

∴ $l = 1 \times 10^6 / 20 \times 10^3 = 50$ mm **Ans.**

Note : The dimensions of the key may also be obtained from the following relations :

$$w = \frac{d_s}{4} + 3 \text{ mm}; \quad \text{and} \quad t_1 = \frac{w}{2}$$

Design for lever

Let t_2 = Thickness of lever in mm, and

B = Width of the lever in mm.

It is assumed that the lever extends up to the centre of the fulcrum. This assumption results in a slightly stronger lever. Neglecting the effect of T_2 on the lever, the maximum bending moment at the

Contents

centre of the fulcrum,

$$M = P \times l = 863.5 \times 500 = 431\ 750 \text{ N-mm}$$

and section modulus, $Z = \frac{1}{6} t_2.B^2 = \frac{1}{6} t_2 \ (2t_2)^2 = 0.67 \ (t_2)^3$ ...(Assuming $B = 2t_2$)

We know that bending tensile stress (σ_t),

$$70 = \frac{M}{Z} = \frac{431\ 750}{0.67\ (t_2)^3} = \frac{644\ 400}{(t_2)^3}$$

$\therefore$ $(t_2)^3 = 644\ 400 / 70 = 9206$ or $t_2 = 21$ mm **Ans.**

and $B = 2\ t_2 = 2 \times 21 = 42$ mm **Ans.**

Design for fulcrum pin

Let d_1 = Diameter of the fulcrum pin, and

l_1 = Length of the fulcrum pin.

First of all, let us find the resultant force acting on the pin. Resolving the three forces T_1, T_2 and P into their vertical and horizontal components, as shown in Fig. 25.30.

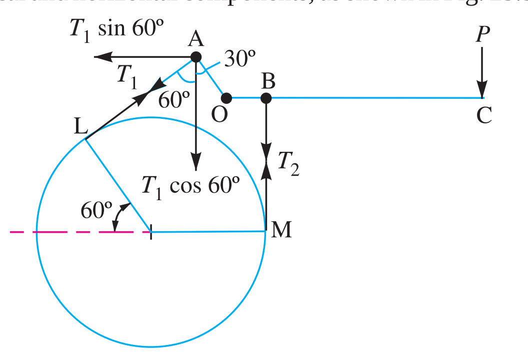
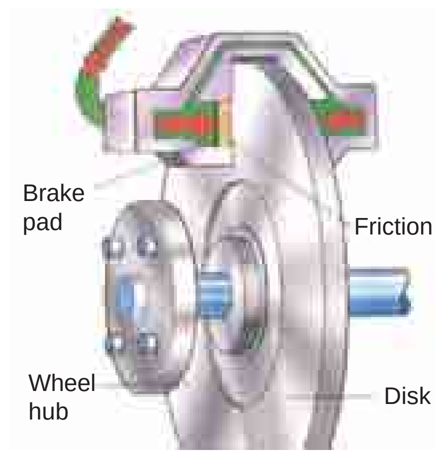


Fig. 25.30



Another type brake disc

We know that sum of vertical components,

$$\Sigma V = T_1 \cos 60° + T_2 + P = 5587 \times \frac{1}{2} + 1587 + 863.5 = 5244 \text{ N}$$

and sum of horizontal components,

$$\Sigma H = T_1 \sin 60° = 5587 \times 0.866 = 4838 \text{ N}$$

$\therefore$ Resultant force acting on pin,

$$R_P = \sqrt{(\Sigma V)^2 + (\Sigma H)^2} = \sqrt{(5244)^2 + (4838)^2} = 7135 \text{ N}$$

Considering bearing of the pin, we have resultant force on the pin (R_P),

$7\ 135 = d_1.l_1.\sigma_b = d_1 \times 1.25\ d_1 \times 20 = 25\ (d_1)^2$ ...(Assuming $l_1 = 1.25\ d_1$)

$\therefore$ $(d_1)^2 = 7135 / 25 = 285.4$ or $d_1 = 16.9$ say 18 mm **Ans.**

and $l_1 = 1.25\ d_1 = 1.25 \times 18 = 22.5$ mm **Ans.**

Let us now check the pin for induced shear stress. Since the pin is in double shear, therefore resultant force on the pin (R_P),

$$7135 = 2 \times \frac{\pi}{4} (d_1)^2\ \tau = 2 \times \frac{\pi}{4} (18)^2\ \tau = 509\ \tau$$

$\therefore$ $\tau = 7135 / 509 = 14 \text{ N/mm}^2$

Top

This induced shear stress is within permissible limits.

The pin may be checked for induced bending stress. We know that maximum bending moment,

$$M = \frac{5}{24} \times W.l_1 = \frac{5}{24} \times 7135 \times 22.5 = 33\,445 \text{ N-mm}$$

...(Here $W = R_P = 7135$ N)

and section modulus, $$Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32}(18)^3 = 573 \text{ mm}^3$$

$\therefore$ Bending stress induced

$$= \frac{M}{Z} = \frac{33\,445}{573} = 58.4 \text{ N/mm}^2$$

This induced bending stress in the pin is within safe limit of 70 N/mm^2.

The lever has an eye hole for the pin and connectors at band have forked end. A brass bush of 3 mm thickness may be provided in the eye of the lever. Therefore, diameter of hole in the lever

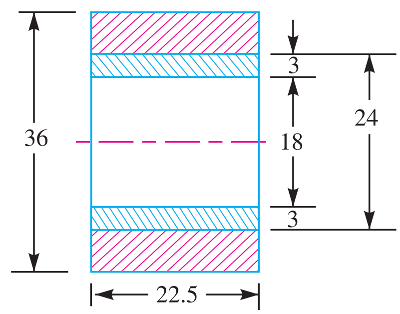
$$= d_1 + 2 \times 3 = 18 + 6 = 24 \text{ mm}$$

The boss is made at the pin joints whose outer diameter is taken equal to twice the diameter of pin and length equal to the length of pin. The inner diameter of boss is equal to the diameter of hole in the lever.

$\therefore$ Outer diameter of boss

$$= 2\,d_1 = 2 \times 18 = 36 \text{ mm}$$

and length of boss $= 22.5$ mm



All dimensions in mm.

Fig. 25.31

Let us now check the induced bending stress in the lever at the fulcrum. The section of the lever at the fulcrum is shown in Fig. 25.31. We know that maximum bending moment at the fulcrum,

$$M = P \times l = 863.5 \times 500 = 431\,750 \text{ N-mm}$$

and section modulus,

$$Z = \frac{\frac{1}{12} \times 22.5\left[(36)^3 - (24)^3\right]}{36/2} = 3420 \text{ mm}^3$$

$\therefore$ Bending stress induced in the lever

$$= \frac{M}{Z} = \frac{431\,750}{3420} = 126 \text{ N/mm}^2$$

Since the induced bending stress is more than the permissible value of 70 N/mm^2, therefore the diameter of pin is required to be increased. Let us take

Diameter of pin, $d_1 = 22$ mm **Ans.**

$\therefore$ Length of pin, $l_1 = 1.25\,d_1 = 1.25 \times 22 = 27.5$ say 28 mm **Ans.**

Diameter of hole in the lever

$$= d_1 + 2 \times 3 = 22 + 6 = 28 \text{ mm}$$

Outer diameter of boss

$= 2\,d_1 = 2 \times 22 = 44$ mm

$=$ Outer diameter of eye

and thickness of each eye $= l_1 / 2 = 28 / 2 = 14$ mm

A clearance of 1.5 mm is provided on either side of the lever in the fork.

The new section of the lever at the fulcrum will be as shown in Fig. 25.32.

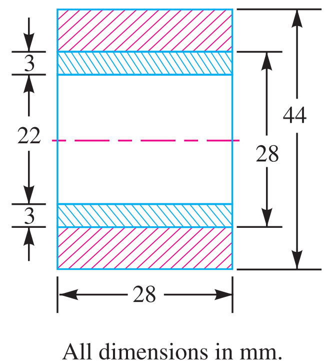


Fig. 25.32

∴ Section modulus,

$$Z = \frac{\frac{1}{12} \times 28\left[(44)^3 - (28)^3\right]}{44/2} = 6706 \text{ mm}^2$$

and induced bending stress $= \dfrac{431\,750}{6706} = 64.4$ N/mm^2

This induced bending stress is within permissible limits.

25.11 Band and Block Brake

The band brake may be lined with blocks of wood or other material, as shown in Fig. 25.33 (*a*). The friction between the blocks and the drum provides braking action. Let there are '*n*' number of blocks, each subtending an angle 2 θ at the centre and the drum rotates in anticlockwise direction.

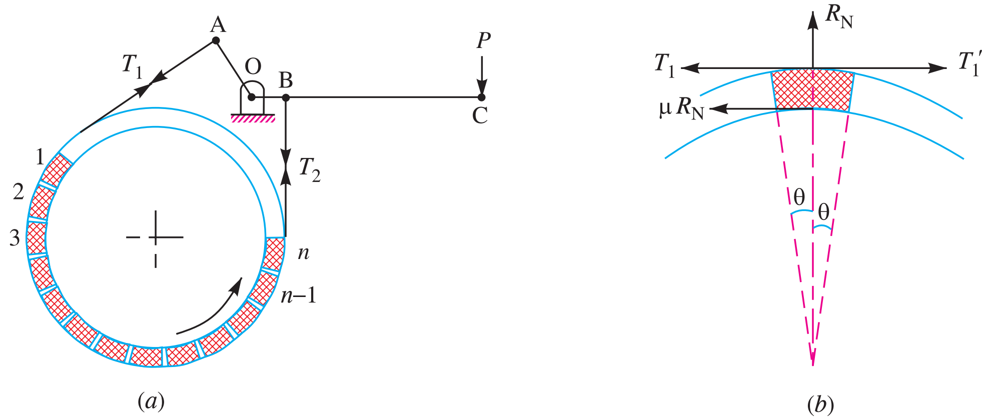


Fig. 25.33. Band and block brake.

Let T_1 = Tension in the tight side,

T_2 = Tension in the slack side,

μ = Coefficient of friction between the blocks and drum,

T_1' = Tension in the band between the first and second block,

T_2', T_3' etc. = Tensions in the band between the second and third block, between the third and fourth block etc.

Consider one of the blocks (say first block) as shown in Fig. 25.33 (*b*). This is in equilibrium under the action of the following forces :

1. Tension in the tight side (T_1),
2. Tension in the slack side (T_1') or tension in the band between the first and second block,
3. Normal reaction of the drum on the block (R_N), and
4. The force of friction ($\mu . R_N$).

Resolving the forces radially, we have

$$(T_1 + T_1') \sin\theta = R_N \quad \text{...}(i)$$

Resolving the forces tangentially, we have

$$(T_1 - T_1') \cos\theta = \mu.R_N \quad \text{...}(ii)$$

Dividing equation (*ii*) by (*i*), we have

$$\frac{(T_1 - T_1')\cos\theta}{(T_1 + T_1')\sin\theta} = \frac{\mu.R_N}{R_N}$$

or

$$(T_1 - T_1') = \mu \tan\theta\,(T_1 + T_1')$$

$$\therefore \quad \frac{T_1}{T_1'} = \frac{1 + \mu\tan\theta}{1 - \mu\tan\theta}$$

Similarly it can be proved for each of the blocks that

$$\frac{T_1'}{T_2'} = \frac{T_2'}{T_3'} = \frac{T_3'}{T_4'} = \ldots = \frac{T_{n-1}}{T_2} = \frac{1 + \mu\tan\theta}{1 - \mu\tan\theta}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{T_1}{T_1'} \times \frac{T_1'}{T_2'} \times \frac{T_2'}{T_3'} \times \ldots \times \frac{T_{n-1}}{T_2} = \left(\frac{1 + \mu\tan\theta}{1 - \mu\tan\theta}\right)^n \quad \text{...}(iii)$$

Braking torque on the drum of effective radius r_e,

$$T_B = (T_1 - T_2)\, r_e$$

$$= (T_1 - T_2)\, r \quad \text{...(Neglecting thickness of band)}$$

Note: For the first block, the tension in the tight side is T_1 and in the slack side is T_1' and for the second block, the tension in the tight side is T_1' and in the slack side is T_2'. Similarly for the third block, the tension in the tight side is T_2' and in the slack side is T_3' and so on. For the last block, the tension in the tight side is T_{n-1} and in the slack side is T_2.

Example 25.14. *In the band and block brake shown in Fig. 25.34, the band is lined with 12 blocks each of which subtends an angle of 15° at the centre of the rotating drum. The thickness of the blocks is 75 mm and the diameter of the drum is 850 mm. If, when the brake is inaction, the greatest and least tensions in the brake strap are T_1 and T_2, show that*

$$\frac{T_1}{T_2} = \left(\frac{l + \mu\tan 7\,{}^1/_2{}^\circ}{l - \mu\tan 7\,{}^1/_2{}^\circ}\right)^{12}$$

where μ is the coefficient of friction for the blocks.

With the lever arrangement as shown in Fig. 25.34, find the least force required at C for the blocks to absorb 225 kW at 240 r.p.m. The coefficient of friction between the band and blocks is 0.4.

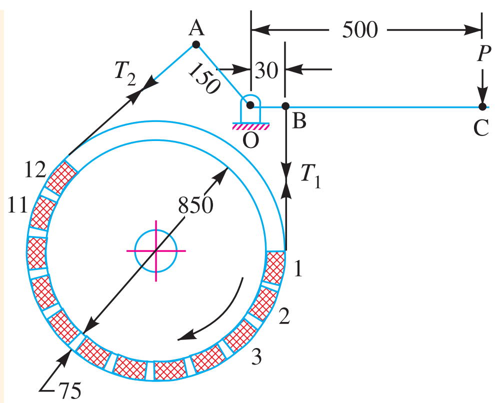


Fig. 25.34

Solution. Given : $n = 12$; $2\,\theta = 15°$ or $\theta = 7\,{}^1/_2{}°$; $t = 75$ mm = 0.075 m ; $d = 850$ mm = 0.85 m ; Power = 225 kW = 225×10^3 W ; $N = 240$ r.p.m. ; $\mu = 0.4$

Since $OA > OB$, therefore the force at C must act downward. Also, the drum rotates clockwise, therefore the end of the band attached to A will be slack with tension T_2 (least tension) and the end of the band attached to B will be tight with tension T_1 (greatest tension).

Consider one of the blocks (say first block) as shown is Fig. 25.35. This is in equilibrium under the action of the following four forces :

1. Tension in the tight side (T_1),
2. Tension in the slack side (T_1') or the tension in the band between the first and second block,
3. Normal reaction of the drum on the block (R_N), and
4. The force of friction ($\mu.R_N$).

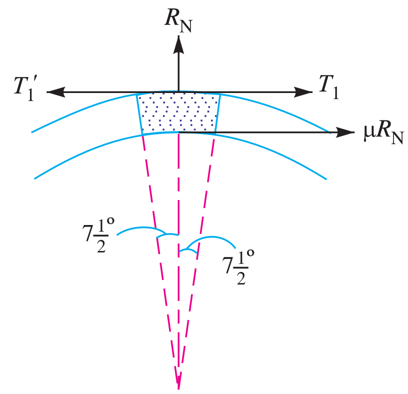


Fig. 25.35

Car wheels are made of alloys to bear high stresses and fatigue.

Resolving the forces radially, we have

$$(T_1 + T_1') \sin 7\tfrac{1}{2}° = R_N \qquad ...(i)$$

Resolving the forces tangentially, we have

$$(T_1 - T_1') \cos 7\tfrac{1}{2}° = \mu.R_N \qquad ...(ii)$$

Dividing equation (*ii*) by (*i*), we have

$$\frac{(T_1 - T_1')\cos 7\tfrac{1}{2}°}{(T_1 + T_1')\sin 7\tfrac{1}{2}°} = \mu \quad \text{or} \quad \frac{T_1 - T_1'}{T_1 + T_1'} = \mu \tan 7\tfrac{1}{2}°$$

$$\therefore \quad \frac{T_1}{T_1'} = \frac{1 + \mu \tan 7\tfrac{1}{2}°}{1 - \mu \tan 7\tfrac{1}{2}°}$$

Similarly, for the other blocks, the ratio of tensions $\frac{T_1'}{T_2'} = \frac{T_2'}{T_3'}$ etc., remains constant. Therefore for 12 blocks having greatest tension T_1 and least tension T_2 is

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan 7\tfrac{1}{2}°}{1 - \mu \tan 7\tfrac{1}{2}°}\right)^{12}$$

Least force required at C

Let P = Least force required at C.

We know that diameter of band,

$$D = d + 2\,t = 0.85 + 2 \times 0.075 = 1 \text{ m}$$

Contents

and power absorbed $= \dfrac{(T_1 - T_2)\pi D.N}{60}$

$$\therefore \quad T_1 - T_2 = \frac{\text{Power} \times 60}{\pi DN} = \frac{225 \times 10^3 \times 60}{\pi \times 1 \times 240} = 17\ 900 \text{ N} \qquad \text{...}(iii)$$

We have proved that

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan 7\frac{1}{2}°}{1 - \mu \tan 7\frac{1}{2}°}\right)^{12} = \left(\frac{1 + 0.4 \times 0.1317}{1 - 0.4 \times 0.1317}\right)^{12} = 3.55 \qquad \text{...}(iv)$$

From equations ***(iii)*** and ***(iv)***, we find that

$$T_1 = 24\ 920 \text{ N} \text{ ; and } T_2 = 7020 \text{ N}$$

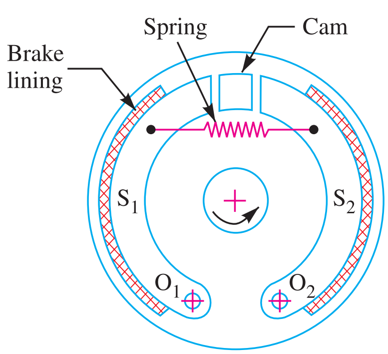
Now taking moments about O, we have

$$P \times 500 = T_2 \times 150 - T_1 \times 30 = 7020 \times 150 - 24\ 920 \times 30 = 305\ 400$$

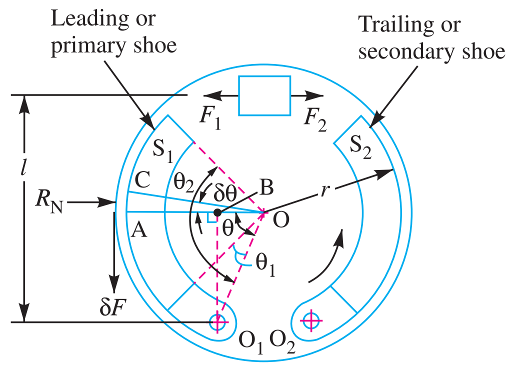
$$\therefore \quad P = 305\ 400 / 500 = 610.8 \text{ N} \quad \textbf{Ans.}$$

25.12 Internal Expanding Brake

An internal expanding brake consists of two shoes S_1 and S_2 as shown in Fig. 25.36 (*a*). The outer surface of the shoes are lined with some friction material (usually with Ferodo) to increase the coefficient of friction and to prevent wearing away of the metal. Each shoe is pivoted at one end about a fixed fulcrum O_1 and O_2 and made to contact a cam at the other end. When the cam rotates, the shoes are pushed outwards against the rim of the drum. The friction between the shoes and the drum produces the braking torque and hence reduces the speed of the drum. The shoes are normally held in off position by a spring as shown in Fig. 25.36 (*a*). The drum encloses the entire mechanism to keep out dust and moisture. This type of brake is commonly used in motor cars and light trucks.



(*a*) Internal expanding brake.



(*b*) Forces on an internal expanding brake.

Fig. 25.36

We shall now consider the forces acting on such a brake, when the drum rotates in the anticlockwise direction as shown in Fig. 25.36 (*b*). It may be noted that for the anticlockwise direction, the left hand shoe is known as ***leading* or *primary shoe*** while the right hand shoe is known as ***trailing* or *secondary shoe.***

Let r = Internal radius of the wheel rim.

b = Width of the brake lining.

p_1 = Maximum intensity of normal pressure,

p_N = Normal pressure,

F_1 = Force exerted by the cam on the leading shoe, and

F_2 = Force exerted by the cam on the trailing shoe.

Top

Consider a small element of the brake lining AC subtending an angle $\delta\theta$ at the centre. Let OA makes an angle θ with OO_1 as shown in Fig. 25.36 (*b*). It is assumed that the pressure distribution on the shoe is nearly uniform, however the friction lining wears out more at the free end. Since the shoe turns about O_1, therefore the rate of wear of the shoe lining at A will be proportional to the radial displacement of that point. The rate of wear of the shoe lining varies directly as the perpendicular distance from O_1 to OA, *i.e.* O_1B. From the geometry of the figure,

Inside view of a truck disk brake

$$O_1B = OO_1 \sin\theta$$

and normal pressure at A, $p_N \propto \sin\theta$ or $p_N = p_1 \sin\theta$

∴ Normal force acting on the element,

$$\delta R_N = \text{Normal pressure} \times \text{Area of the element}$$

$$= p_N\,(b\,.\,r\,.\,\delta\theta) = p_1 \sin\theta\,(b\,.\,r\,.\,\delta\theta)$$

and braking or friction force on the element,

$$\delta F = \mu.\delta R_N = \mu\, p_1 \sin\theta\,(b\,.\,r\,.\,\delta\theta)$$

∴ Braking torque due to the element about O,

$$\delta T_B = \delta F\,.\,r = \mu\, p_1 \sin\theta\,(b\,.\,r\,.\,\delta\theta)\,r = \mu\, p_1\, b\, r^2 (\sin\theta.\delta\theta)$$

and total braking torque about O for whole of one shoe,

$$T_B = \mu\; p_1\; b\; r^2 \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = \mu\; p_1\; b\; r^2\; [-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1\, b\, r^2 (\cos\theta_1 - \cos\theta_2)$$

Moment of normal force δR_N of the element about the fulcrum O_1,

$$\delta M_N = \delta R_N \times O_1B = \delta R_N\,(OO_1 \sin\theta)$$

$$= p_1 \sin\theta\,(b\,.\,r\,.\,\delta\theta)\,(OO_1 \sin\theta) = p_1 \sin^2\theta\,(b\,.\,r\,.\,\delta\theta)\,OO_1$$

Total moment of normal forces about the fulcrum O_1,

$$M_N = \int_{\theta_1}^{\theta_2} p_1 \sin^2\theta\,(b\,.\,r\;\delta\theta)\,OO_1 = p_1\,.\,b\,.\,r\,.\,OO_1 \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta$$

$$= p_1.b.r.OO_1 \int_{\theta_1}^{\theta_2} \tfrac{1}{2}(1-\cos 2\theta)\, d\theta \qquad \ldots\left[\because \sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta)\right]$$

$$= \tfrac{1}{2}\, p_1\,.\,b\,.\,r\,.\,OO_1 \left[\theta - \frac{\sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$$

Contents

$$= \frac{1}{2}\, p_1 \,.\, b \,.\, r \,.\, OO_1 \left[\theta_2 - \frac{\sin 2\theta_2}{2} - \theta_1 + \frac{\sin 2\theta_1}{2}\right]$$

$$= \frac{1}{2}\, p_1 \,.\, b \,.\, r \,.\, OO_1 \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)\right]$$

Moment of frictional force δF about the fulcrum O_1,

$$\delta M_F = \delta F \times AB = \delta F\,(r - OO_1 \cos\theta) \qquad \text{...}(\because AB = r - OO_1 \cos\theta)$$

$$= \mu \,.\, p_1 \sin\theta\,(b \,.\, r \,.\, \delta\theta)\,(r - OO_1 \cos\theta)$$

$$= \mu \,.\, p_1 \,.\, b \,.\, r\,(r \sin\theta - OO_1 \sin\theta\cos\theta)\,\delta\theta$$

$$= \mu \,.\, p_1 \,.\, b \,.\, r \left(r \sin\theta - \frac{OO_1}{2}\sin 2\theta\right)\delta\theta \qquad \text{...}(\because 2\sin\theta\cos\theta = \sin 2\theta)$$

$\therefore$ Total moment of frictional force about the fulcrum O_1,

$$M_F = \mu \,.\, p_1 \,.\, b \,.\, r \int_{\theta_1}^{\theta_2} \left(r \sin\theta - \frac{OO_1}{2}\sin 2\theta\right) d\theta$$

$$= \mu \,.\, p_1 \,.\, b \,.\, r \left[-r\cos\theta + \frac{OO_1}{4}\cos 2\theta\right]_{\theta_1}^{\theta_2}$$

$$= \mu \,.\, p_1 \,.\, b \,.\, r \left[-r\cos\theta_2 + \frac{OO_1}{4}\cos 2\theta_2 + r\cos\theta_1 - \frac{OO_1}{4}\cos 2\theta_1\right]$$

$$= \mu \,.\, p_1 \,.\, b \,.\, r \left[r\,(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$$

Now for leading shoe, taking moments about the fulcrum O_1,

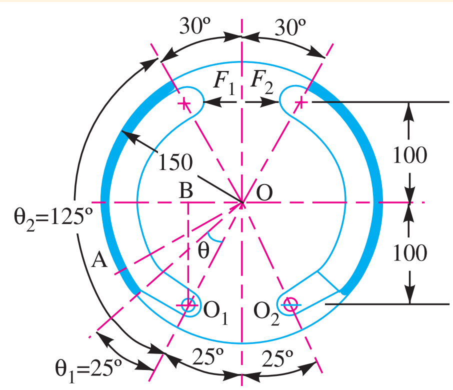
$$F_1 \times l = M_N - M_F$$

and for trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

Note: If $M_F > M_N$, then the brake becomes self locking.

Example 25.15. *Fig. 25.37 shows the arrangement of two brake shoes which act on the internal surface of a cylindrical brake drum. The braking force F_1 and F_2 are applied as shown and each shoe pivots on its fulcrum O_1 and O_2. The width of the brake lining is 35 mm. The intensity of pressure at any point A is 0.4 sin θ N/mm², where θ is measured as shown from either pivot. The coefficient of friction is 0.4. Determine the braking torque and the magnitude of the forces F_1 and F_2.*



All dimensions in mm.

Fig. 25.37

Top

Solution. Given : b = 35 mm ; μ = 0.4 ; r = 150 mm ; l = 200 mm ; θ_1 = 25° ; θ_2 = 125°

Since the intensity of normal pressure at any point is 0.4 sin θ N/mm^2, therefore maximum intensity of normal pressure,

$$p_1 = 0.4 \text{ N/mm}^2$$

We know that the braking torque for one shoe,

$$= \mu \,.\, p_1 \,.\, b \,.\, r^2 (\cos \theta_1 - \cos \theta_2)$$
$$= 0.4 \times 0.4 \times 35\, (150)^2 (\cos 25° - \cos 125°)$$
$$= 126\ 000\, (0.9063 + 0.5736) = 186\ 470 \text{ N-mm}$$

∴ Total braking torque for two shoes,

$$T_B = 2 \times 186\ 470 = 372\ 940 \text{ N-mm}$$

Magnitude of the forces F_1 and F_2

From the geometry of the figure, we find that

$$OO_1 = \frac{O_1 B}{\cos 25°} = \frac{100}{0.9063} = 110.3 \text{ mm}$$

$$\theta_1 = 25° = 25 \times \pi / 180 = 0.436 \text{ rad}$$

and $$\theta_2 = 125° = 125 \times \pi / 180 = 2.18 \text{ rad}$$

We know that the total moment of normal forces about the fulcrum O_1,

$$M_N = \tfrac{1}{2}\, p_1 \,.\, b \,.\, r \,.\, OO_1 \left[(\theta_2 - \theta_1) + \tfrac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)\right]$$
$$= \tfrac{1}{2} \times 0.4 \times 35 \times 150 \times 110.3 \left[(2.18 - 0.436) + \tfrac{1}{2}(\sin 50° - \sin 250°)\right]$$
$$= 115\ 815 \left[1.744 + \frac{1}{2}(0.766 + 0.9397)\right] = 300\ 754 \text{ N-mm}$$

and total moment of friction force about the fulcrum O_1,

$$M_F = \mu \,.\, p_1 \,.\, b \,.\, r \left[r(\cos \theta_1 - \cos \theta_2) + \frac{OO_1}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$$
$$= 0.4 \times 0.4 \times 35 \times 150 \left[150\,(\cos 25° - \cos 125°) + \frac{110.3}{4}(\cos 250° - \cos 50°)\right]$$
$$= 840\, [150\, (0.9063 + 0.5736) + 27.6\, (-0.342 - 0.6428)]$$
$$= 840\, (222 - 27) = 163\ 800 \text{ N-mm}$$

For the leading shoe, taking moments about the fulcrum O_1,

$$F_1 \times l = M_N - M_F$$

or $$F_1 \times 200 = 300\ 754 - 163\ 800 = 136\ 954$$

∴ $$F_1 = 136\ 954 / 200 = 685 \text{ N}$$ **Ans.**

For the trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

or $$F_2 \times 200 = 300\ 754 + 163\ 800 = 464\ 554$$

∴ $$F_2 = 464\ 554 / 200 = 2323 \text{ N}$$ **Ans.**

EXERCISES

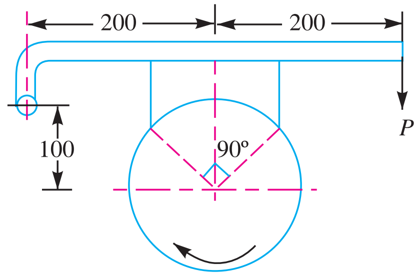
1. A flywheel of mass 100 kg and radius of gyration 350 mm is rotating at 720 r.p.m. It is brought to rest by means of a brake. The mass of the brake drum assembly is 5 kg. The brake drum is made of cast iron FG 260 having specific heat 460 J / kg°C. Assuming that the total heat generated is absorbed by the brake drum only, calculate the temperature rise.

[Ans. 15.14°C]

Contents

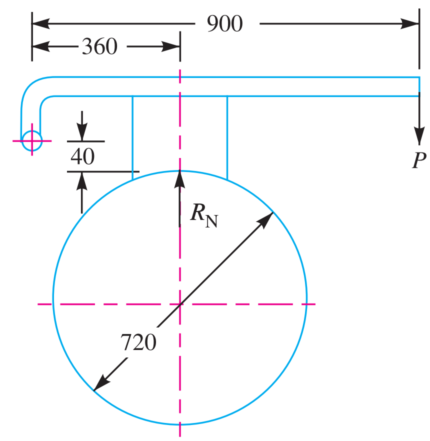
2. A single block brake, as shown in Fig. 25.38, has the drum diameter 250 mm. The angle of contact is 90° and the coefficient of friction between the drum and the lining is 0.35. If the torque transmitted by the brake is 70 N-m, find the force *P* required to operate the brake.

[Ans. 700 N]



All dimensions in mm.

Fig. 25.38



All dimensions in mm.

Fig. 25.39

3. A single block brake, as shown in Fig. 25.39, has a drum diameter of 720 mm. If the brake sustains 225 N-m torque at 500 r.p.m.; find :

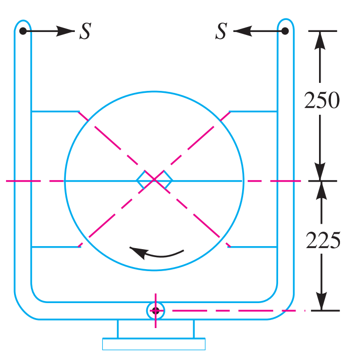
   (*a*) the required force (*P*) to apply the brake for clockwise rotation of the drum;

   (*b*) the required force (*P*) to apply the brake for counter clockwise rotation of the drum;

   (*c*) the location of the fulcrum to make the brake self-locking for clockwise rotation of the drum; and

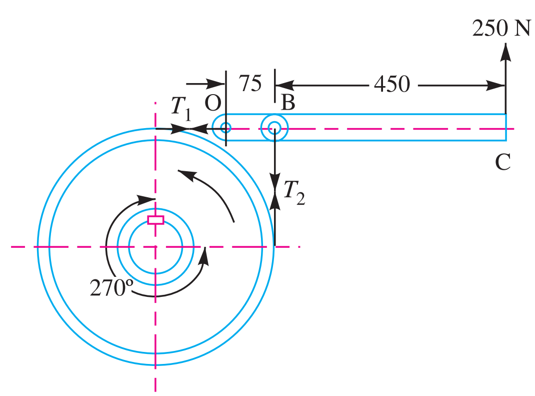
   The coefficient of friction may be taken as 0.3. **[Ans. 805.4 N ; 861 N; 1.2 m ; 11.78 kW]**

4. The layout and dimensions of a double shoe brake is shown in Fig. 25.40. The diameter of the brake drum is 300 mm and the contact angle for each shoe is 90°. If the coefficient of friction for the brake lining and the drum is 0.4, find the spring force necessary to transmit a torque of 30 N-m. Also determine the width of the brake shoes, if the bearing pressure on the lining material is not to exceed 0.28 N/mm^2. **[Ans. 99.1 N ; 5 mm]**



All dimensions in mm.

Fig. 25.40



All dimensions in mm.

Fig. 25.41

Top

5. The drum of a simple band brake is 450 mm. The band embraces 3/4th of the circumference of the drum. One end of the band is attached to the fulcrum pin and the other end is attached to a pin *B* as shown in Fig. 25.41. The band is to be lined with asbestos fabric having a coefficient of friction 0.3. The allowable bearing pressure for the brake lining is 0.21 N/mm^2. Design the band shaft, key, lever and fulcrum pin. The material of these parts is mild steel having permissible stresses as follows :

$$\sigma_t = \sigma_c = 70 \text{ MPa, and } \tau = 56 \text{ MPa}$$

6. A band brake as shown in Fig. 25.42, is required to balance a torque of 980 N-m at the drum shaft. The drum is to be made of 400 mm diameter and is keyed to the shaft. The band is to be lined with ferodo lining having a coefficient of friction 0.25. The maximum pressure between the lining and drum is 0.5 N/mm^2. Design the steel band, shaft, key on the shaft, brake lever and fulcrum pin. The permissible stresses for the steel to be used for the shaft, key, band lever and pin are 70 MPa in tension and compression and 56 MPa in shear.

7. A differential band brake is shown in Fig. 25.43. The diameter of the drum is 800 mm. The coefficient of friction between the band and the drum is 0.3 and the angle of embrace is 240°. When a force of 600 N

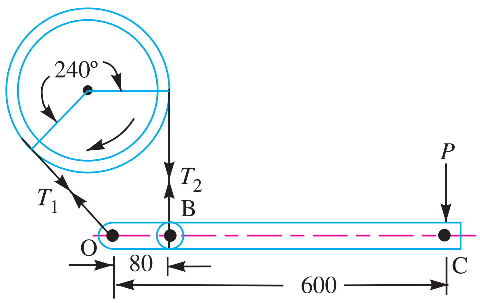


Fig. 25.42

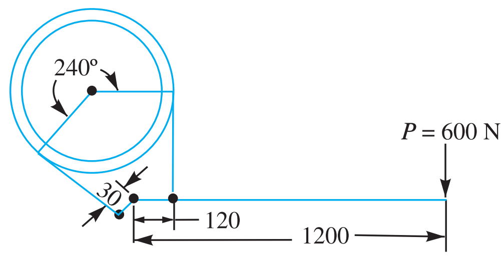


Fig. 25.43

is applied at the free end of the lever, find for the clockwise and anticlockwise rotation of the drum: 1. the maximum and minimum forces in the band; and 2. the torque which can be applied by the brake.

[Ans. 176 kN, 50 kN, 50.4 kN-m ; 6.46 kN, 1.835 kN, 1.85 kN-m]

8. In a band and block brake, the band is lined with 14 blocks, each of which subtends an angle of 20° at the drum centre. One end of the band is attached to the fulcrum of the brake lever and the other to a pin 150 mm from the fulcrum. Find the force required at the end of the lever 1 metre long from the fulcrum to give a torque of 4 kN-m. The diameter of the brake drum is 1 metre and the coefficient of friction between the blocks and the drum is 0.25. **[Ans. 1692 N]**

QUESTIONS

1. How does the function of a brake differ from that of a clutch ?
2. A weight is brought to rest by applying brakes to the hoisting drum driven by an electric motor. How will you estimate the total energy absorbed by the brake ?
3. What are the thermal considerations in brake design ?
4. What is the significance of *pV* value in brake design ?
5. What are the materials used for brake linings.
6. Discuss the different types of brakes giving atleast one practical application for each.
7. List the important factors upon which the capacity of a brake depends.
8. What is a self-energizing brake ? When a brake becomes self-locking.

Contents

9. What is back stop action in band brakes ? Explain the condition for it.

10. Describe with the help of a neat sketch the principle of operation of an internal expanding shoe brake. Derive the expression for the braking torque.

Truck suspension system : Front Pivot ball suspension soaks up the bumps and provides unmatched adjustability. Chrome 8 mm CVA joints give added strength.

Note : This picture is given as additional information and is not a direct example of the current chapter.

OBJECTIVE TYPE QUESTIONS

1. A brake commonly used in railway trains is
   (*a*) shoe brake (*b*) band brake
   (*c*) band and block brake (*d*) internal expanding brake
2. A brake commonly used in motor cars is
   (*a*) shoe brake (*b*) band brake
   (*c*) band and block brake (*d*) internal expanding brake
3. The material used for brake lining should have .... coefficient of friction.
   (*a*) low (*b*) high
4. When the frictional force helps to apply the brake, then the brake is said to be
   (*a*) self-energizing brake (*b*) self-locking brake
5. For a band brake, the width of the band for a drum diameter greater than 1 m, should not exceed
   (*a*) 150 mm (*b*) 200 mm
   (*c*) 250 mm (*d*) 300 mm

ANSWERS

1. (*a*) **2.** (*d*) **3.** (*b*) **4.** (*a*) **5.** (*a*)

Top