1

Introduction: Dimensional Analysis—Basic Thermodynamics and Fluid Mechanics

1.1 INTRODUCTION TO TURBOMACHINERY

A turbomachine is a device in which energy transfer occurs between a flowing fluid and a rotating element due to dynamic action, and results in a change in pressure and momentum of the fluid. Mechanical energy transfer occurs inside or outside of the turbomachine, usually in a steady-flow process. Turbomachines include all those machines that produce power, such as turbines, as well as those types that produce a head or pressure, such as centrifugal pumps and compressors. The turbomachine extracts energy from or imparts energy to a continuously moving stream of fluid. However in a positive displacement machine, it is intermittent.

The turbomachine as described above covers a wide range of machines, such as gas turbines, steam turbines, centrifugal pumps, centrifugal and axial flow compressors, windmills, water wheels, and hydraulic turbines. In this text, we shall deal with incompressible and compressible fluid flow machines.

1.2 TYPES OF TURBOMACHINES

There are different types of turbomachines. They can be classified as:

1. Turbomachines in which (i) work is done by the fluid and (ii) work is done on the fluid.

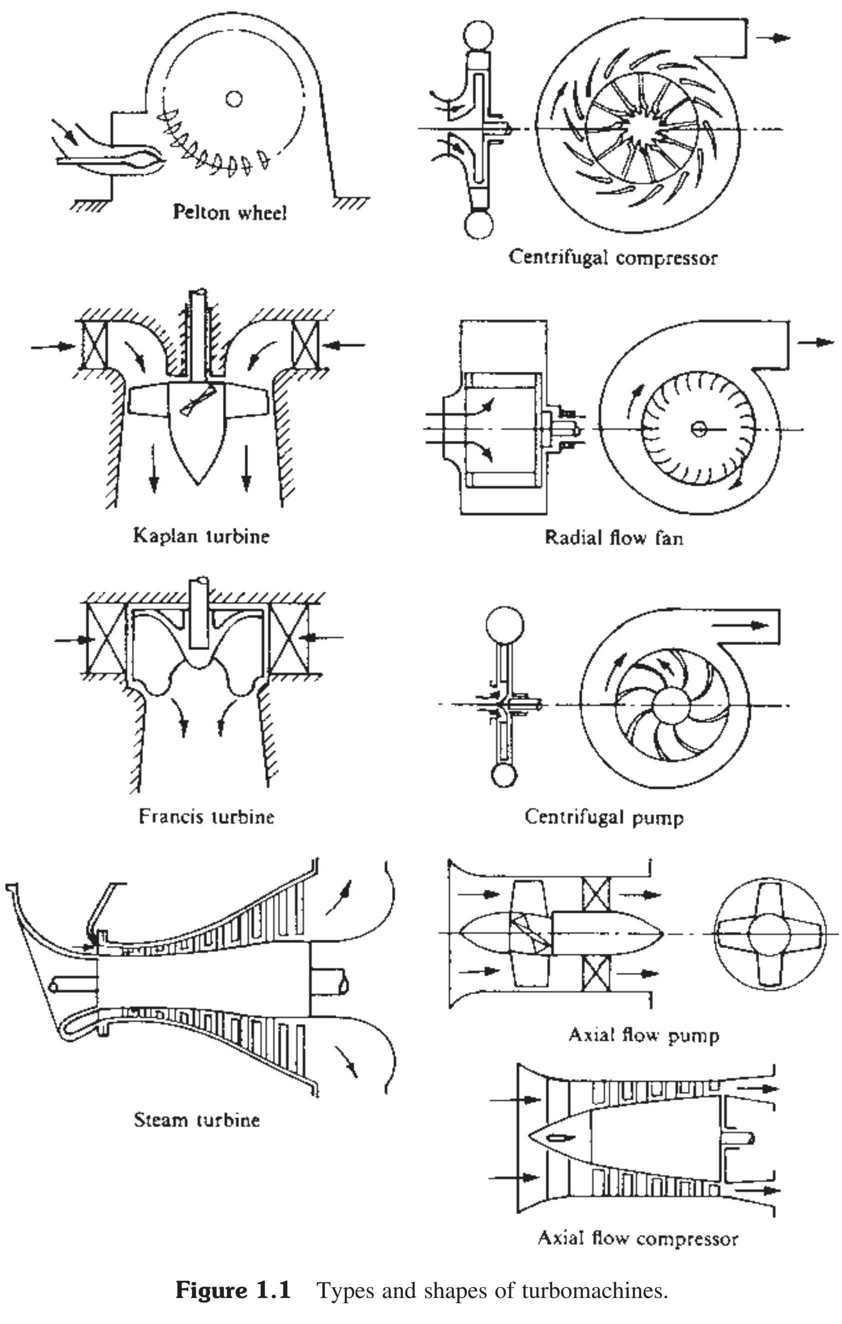

Figure 1.1 Types and shapes of turbomachines.

2. Turbomachines in which fluid moves through the rotating member in axial direction with no radial movement of the streamlines. Such machines are called axial flow machines whereas if the flow is essentially radial, it is called a radial flow or centrifugal flow machine. Some of these machines are shown in Fig. 1.1, and photographs of actual machines are shown in Figs. 1.2–1.6. Two primary points will be observed: first, that the main element is a rotor or runner carrying blades or vanes; and secondly, that the path of the fluid in the rotor may be substantially axial, substantially radial, or in some cases a combination of both. Turbomachines can further be classified as follows:

   Turbines: Machines that produce power by expansion of a continuously flowing fluid to a lower pressure or head.

   Pumps: Machines that increase the pressure or head of flowing fluid.

   Fans: Machines that impart only a small pressure-rise to a continuously flowing gas; usually the gas may be considered to be incompressible.

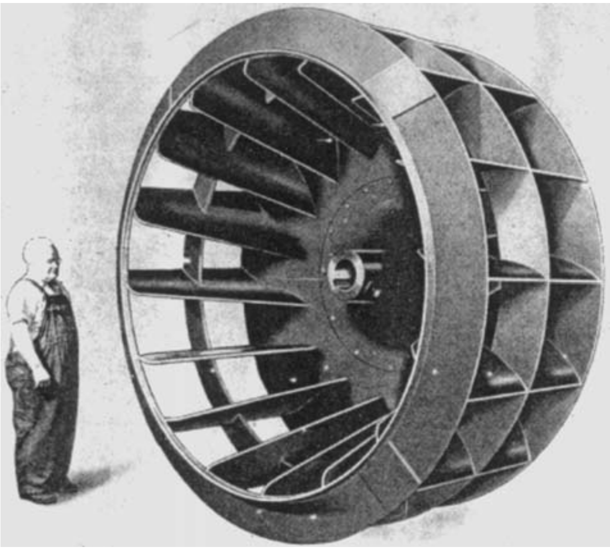

Figure 1.2 Radial flow fan rotor. (Courtesy of the Buffalo Forge Corp.)

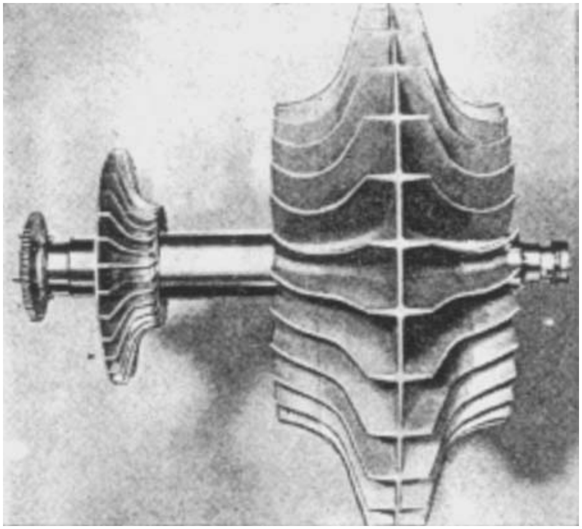

Figure 1.3 Centrifugal compressor rotor (the large double-sided impellar on the right is the main compressor and the small single-sided impellar is an auxiliary for cooling purposes). (Courtesy of Rolls-Royce, Ltd.)

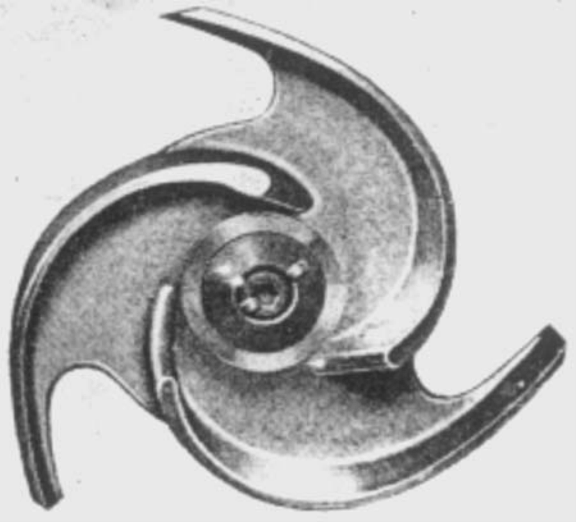

Figure 1.4 Centrifugal pump rotor (open type impeller). (Courtesy of the Ingersoll-Rand Co.)

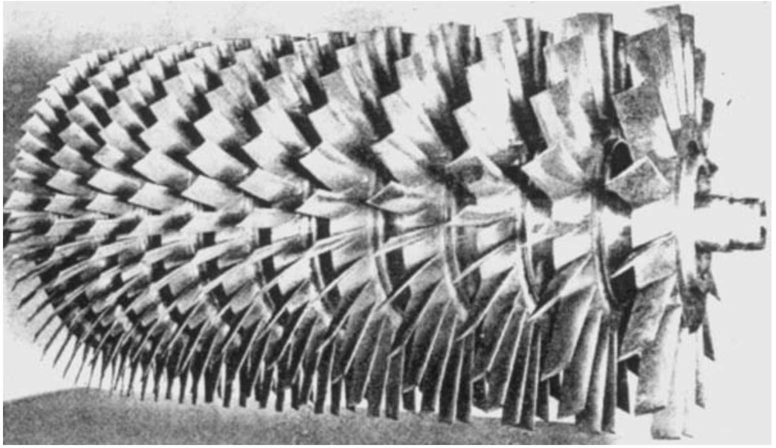

Figure 1.5 Multi-stage axial flow compressor rotor. (Courtesy of the Westinghouse Electric Corp.)

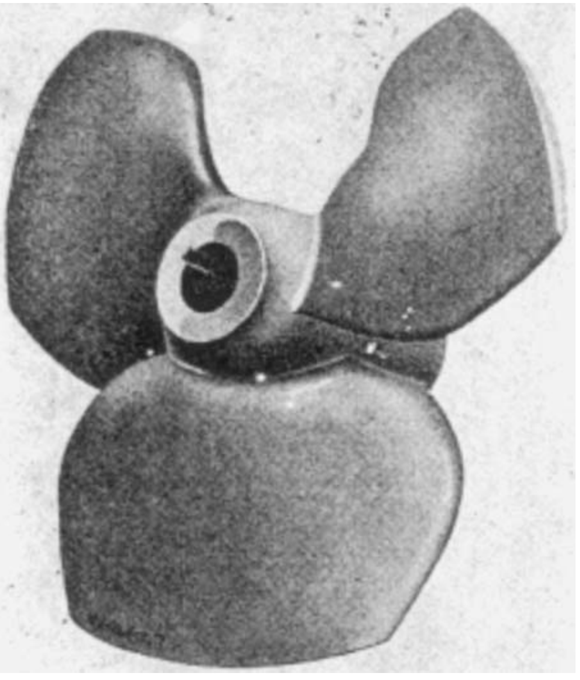

Figure 1.6 Axial flow pump rotor. (Courtesy of the Worthington Corp.)

Compressors: Machines that impart kinetic energy to a gas by compressing it and then allowing it to rapidly expand. Compressors can be axial flow, centrifugal, or a combination of both types, in order to produce the highly compressed air. In a dynamic compressor, this is achieved by imparting kinetic energy to the air in the impeller and then this kinetic energy is converted into pressure energy in the diffuser.

1.3 DIMENSIONAL ANALYSIS

To study the performance characteristics of turbomachines, a large number of variables are involved. The use of dimensional analysis reduces the variables to a number of manageable dimensional groups. Usually, the properties of interest in regard to turbomachine are the power output, the efficiency, and the head. The performance of turbomachines depends on one or more of several variables. A summary of the physical properties and dimensions is given in Table 1.1 for reference.

Dimensional analysis applied to turbomachines has two more important uses: (1) prediction of a prototype's performance from tests conducted on a scale

Table 1.1 Physical Properties and Dimensions

Property	Dimension
Surface	L^2
Volume	L^3
Density	M/L^3
Velocity	L/T
Acceleration	L/T^2
Momentum	ML/T
Force	ML/T^2
Energy and work	ML^2/T^2
Power	ML^2/T^3
Moment of inertia	ML^2
Angular velocity	I/T
Angular acceleration	I/T^2
Angular momentum	ML^2/T
Torque	ML^2/T^2
Modules of elasticity	M/LT^2
Surface tension	M/T^2
Viscosity (absolute)	M/LT
Viscosity (kinematic)	L^2/T

model (similitude), and (2) determination of the most suitable type of machine, on the basis of maximum efficiency, for a specified range of head, speed, and flow rate. It is assumed here that the student has acquired the basic techniques of forming nondimensional groups.

1.4 DIMENSIONS AND EQUATIONS

The variables involved in engineering are expressed in terms of a limited number of basic dimensions. For most engineering problems, the basic dimensions are:

1. *SI system*: mass, length, temperature and time.
2. *English system*: mass, length, temperature, time and force.

The dimensions of pressure can be designated as follows

$$P = \frac{F}{L^2} \tag{1.1}$$

Equation (1.1) reads as follows: "The dimension of P equals force per length squared." In this case, L^2 represents the dimensional characteristics of area. The left hand side of Eq. (1.1) must have the same dimensions as the right hand side.

1.5 THE BUCKINGHAM Π THEOREM

In 1915, Buckingham showed that the number of independent dimensionless group of variables (dimensionless parameters) needed to correlate the unknown variables in a given process is equal to $n - m$, where n is the number of variables involved and m is the number of dimensionless parameters included in the variables. Suppose, for example, the drag force F of a flowing fluid past a sphere is known to be a function of the velocity (v) mass density (ρ) viscosity (μ) and diameter (D). Then we have five variables (F, v, ρ, μ, and D) and three basic dimensions (L, F, and T) involved. Then, there are $5 - 3 = 2$ basic grouping of variables that can be used to correlate experimental results.

1.6 HYDRAULIC MACHINES

Consider a control volume around the pump through which an incompressible fluid of density ρ flows at a volume flow rate of Q.

Since the flow enters at one point and leaves at another point the volume flow rate Q can be independently adjusted by means of a throttle valve. The discharge Q of a pump is given by

$$Q = f(N, D, g, H, \mu, \rho) \tag{1.2}$$

where H is the head, D is the diameter of impeller, g is the acceleration due to gravity, ρ is the density of fluid, N is the revolution, and μ is the viscosity of fluid.

In Eq. (1.2), primary dimensions are only four. Taking N, D, and ρ as repeating variables, we get

$$\Pi_1 = (N)^{\mathrm{a}}(D)^{\mathrm{b}}(\rho)^{\mathrm{c}}(Q)$$

$$\mathrm{M}^0\mathrm{L}^0\mathrm{T}^0 = (\mathrm{T}^{-1})^{\mathrm{a}}(\mathrm{L})^{\mathrm{b}}(\mathrm{ML}^{-3})^{\mathrm{c}}(\mathrm{L}^3\mathrm{T}^{-1})$$

For dimensional homogeneity, equating the powers of M, L, and T on both sides of the equation: for M, 0 = c or c = 0; for T, 0 = − a − 1 or a = − 1; for L, 0 = b − 3c + 3 or b = − 3.
Therefore,

$$\Pi_1 = N^{-1}D^{-3}\rho^0 Q = \frac{Q}{ND^3} \tag{1.3}$$

Similarly,

$$\Pi_2 = (N)^{\mathrm{d}}(D)^{\mathrm{e}}(\rho)^{\mathrm{f}}(g)$$

$$\mathrm{M}^0\mathrm{L}^0\mathrm{T}^0 = (\mathrm{T}^{-1})^{\mathrm{d}}(\mathrm{L})^{\mathrm{e}}(\mathrm{ML}^{-3})^{\mathrm{f}}(\mathrm{LT}^{-2})$$

Now, equating the exponents: for M, 0 = f or f = 0; for T, 0 = − d − 2 or d = − 2; for L, 0 = e − 3f + 1 or e = − 1.
Thus,

$$\Pi_2 = N^{-2}D^{-1}\rho^0 g = \frac{g}{N^2 D} \tag{1.4}$$

Similarly,

$$\Pi_3 = (N)^{g}(D)^{h}(\rho)^{i}(H)$$

$$\mathrm{M}^0\mathrm{L}^0\mathrm{T}^0 = (\mathrm{T}^{-1})^{g}(\mathrm{L})^{h}(\mathrm{ML}^{-3})^{i}(\mathrm{L})$$

Equating the exponents: for M, 0 = i or $i = 0$; for T, $0 = -g$ or $g = 0$; for L, 0 = h − 3i + 1 or h = − 1.
Thus,

$$\Pi_3 = N^0 D^{-1}\rho^0 H = \frac{H}{D} \tag{1.5}$$

and,

$$\Pi_4 = (N)^{j}(D)^{k}(\rho)^{l}(\mu)$$

$$\mathrm{M}^0\mathrm{L}^0\mathrm{T}^0 = (\mathrm{T}^{-1})^{j}(\mathrm{L})^{k}(\mathrm{ML}^{-3})^{l}(\mathrm{ML}^{-1}\mathrm{T}^{-1})$$

Equating the exponents: for M, $0 = l + 1$ or $l = -1$; for T, $0 = -j - 1$ or $j = -1$; for L, $0 = k\text{-}3l - 1$ or $k = -2$.

Thus,

$$\Pi_4 = N^{-1}D^{-2}\rho^{-1}\mu = \frac{\mu}{ND^2\rho} \tag{1.6}$$

The functional relationship may be written as

$$f\left(\frac{Q}{ND^3}, \frac{g}{N^2D}, \frac{H}{D}, \frac{\mu}{ND^2\rho}\right) = 0$$

Since the product of two Π terms is dimensionless, therefore replace the terms Π_2 and Π_3 by gh/N^2D^2

$$f\left(\frac{Q}{ND^3}, \frac{gH}{N^2D^2}, \frac{\mu}{ND^2\rho}\right) = 0$$

or

$$Q = ND^3 f\left(\frac{gH}{N^2D^2}, \frac{\mu}{ND^2\rho}\right) = 0 \tag{1.7}$$

A dimensionless term of extremely great importance that may be obtained by manipulating the discharge and head coefficients is the specific speed, defined by the equation

$$N_s = \sqrt{\frac{\text{Flow coefficient}}{\text{Head coefficient}}} = N\sqrt{Q}/(gH)^{3/4} \tag{1.8}$$

The following few dimensionless terms are useful in the analysis of incompressible fluid flow machines:

1. *The flow coefficient and speed ratio*: The term $Q/(ND^3)$ is called the flow coefficient or specific capacity and indicates the volume flow rate of fluid through a turbomachine of unit diameter runner, operating at unit speed. It is constant for similar rotors.
2. *The head coefficient*: The term gH/N^2D^2 is called the specific head. It is the kinetic energy of the fluid spouting under the head H divided by the kinetic energy of the fluid running at the rotor tangential speed. It is constant for similar impellers.

$$\psi = H/(U^2/g) = gH/(\pi^2N^2D^2) \tag{1.9}$$

3. *Power coefficient or specific power*: The dimensionless quantity $P/(\rho N^2D^2)$ is called the power coefficient or the specific power. It shows the relation between power, fluid density, speed and wheel diameter.
4. *Specific speed*: The most important parameter of incompressible fluid flow machinery is specific speed. It is the non-dimensional term. All turbomachineries operating under the same conditions of flow and head

having the same specific speed, irrespective of the actual physical size of the machines. Specific speed can be expressed in this form

$$N_s = N\sqrt{Q}/(gH)^{3/4} = N\sqrt{P}/\lfloor \rho^{1/2}(gH)^{5/4}\rfloor \tag{1.10}$$

The specific speed parameter expressing the variation of all the variables N, Q and H or N,P and H, which cause similar flows in turbomachines that are geometrically similar. The specific speed represented by Eq. (1.10) is a nondimensional quantity. It can also be expressed in alternate forms.
These are

$$N_s = N\sqrt{Q}/H^{3/4} \tag{1.11}$$

and

$$N_s = N\sqrt{P}/H^{5/4} \tag{1.12}$$

Equation (1.11) is used for specifying the specific speeds of pumps and Eq. (1.12) is used for the specific speeds of turbines. The turbine specific speed may be defined as the speed of a geometrically similar turbine, which develops 1 hp under a head of 1 meter of water. It is clear that N_s is a dimensional quantity. In metric units, it varies between 4 (for very high head Pelton wheel) and 1000 (for the low-head propeller on Kaplan turbines).

1.7 THE REYNOLDS NUMBER

Reynolds number is represented by

$$Re = D^2N/\nu$$

where υ is the kinematic viscosity of the fluid. Since the quantity D^2N is proportional to DV for similar machines that have the same speed ratio. In flow through turbomachines, however, the dimensionless parameter D^2N/ν is not as important since the viscous resistance alone does not determine the machine losses. Various other losses such as those due to shock at entry, impact, turbulence, and leakage affect the machine characteristics along with various friction losses.

Consider a control volume around a hydraulic turbine through which an incompressible fluid of density ρ flows at a volume flow rate of Q, which is controlled by a valve. The head difference across the control volume is H, and if the control volume represents a turbine of diameter D, the turbine develops a shaft power P at a speed of rotation N. The functional equation may be written as

$$P = f(\rho, N, \mu, D, Q, gH) \tag{1.13}$$

Equation (1.13) may be written as the product of all the variables raised to a power and a constant, such that

$$P = \text{const.}\left(\rho^a N^b \mu^c D^d Q^e (gH)^f\right) \tag{1.14}$$

Substituting the respective dimensions in the above Eq. (1.14),

$$\left(\mathrm{ML}^2/\mathrm{T}^3\right) = \text{const.}(\mathrm{M/L}^3)^a(1/\mathrm{T})^b(\mathrm{M/LT})^c(\mathrm{L})^d(\mathrm{L}^3/\mathrm{T})^e(\mathrm{L}^2/\mathrm{T}^2)^f \tag{1.15}$$

Equating the powers of M, L, and T on both sides of the equation: for M, $1 = a + c$; for L, $2 = -3a - c + d + 3e + 2f$; for T, $-3 = -b - c - e - 2f$.

There are six variables and only three equations. It is therefore necessary to solve for three of the indices in terms of the remaining three. Solving for a, b, and d in terms of c, e, and f we have:

$$a = 1 - c$$

$$b = 3 - c - \mathrm{e} - 2f$$

$$d = 5 - 2c - 3e - 2f$$

Substituting the values of a, b, and d in Eq. (1.13), and collecting like indices into separate brackets,

$$P = \text{const.}\left[\left(\rho N^3 D^5\right), \left(\frac{\mu}{\rho N D^2}\right)^c, \left(\frac{Q}{ND^3}\right)^e, \left(\frac{gH}{N^2 D^2}\right)^f\right] \tag{1.16}$$

In Eq. (1.16), the second term in the brackets is the inverse of the Reynolds number. Since the value of c is unknown, this term can be inverted and Eq. (1.16) may be written as

$$P/\rho N^3 D^5 = \text{const.}\left[\left(\frac{\rho N D^2}{\mu}\right)^c, \left(\frac{Q}{ND^3}\right)^{\mathrm{e}}, \left(\frac{gH}{N^2 D^2}\right)^f\right] \tag{1.17}$$

In Eq. (1.17) each group of variables is dimensionless and all are used in hydraulic turbomachinery practice, and are known by the following names: the power coefficient $\left(P/\rho N^3 D^5 = \overline{P}\right)$; the flow coefficient $\left(Q/ND^3 = \phi\right)$; and the head coefficient $\left(gH/N^2 D^2 = \psi\right)$.

Eqution (1.17) can be expressed in the following form:

$$\overline{P} = f\left(Re, \phi, \psi\right) \tag{1.18}$$

Equation (1.18) indicates that the power coefficient of a hydraulic machine is a function of Reynolds number, flow coefficient and head coefficient. In flow through hydraulic turbomachinery, Reynolds number is usually very high. Therefore the viscous action of the fluid has very little effect on the power output of the machine and the power coefficient remains only a function of ϕ and ψ.

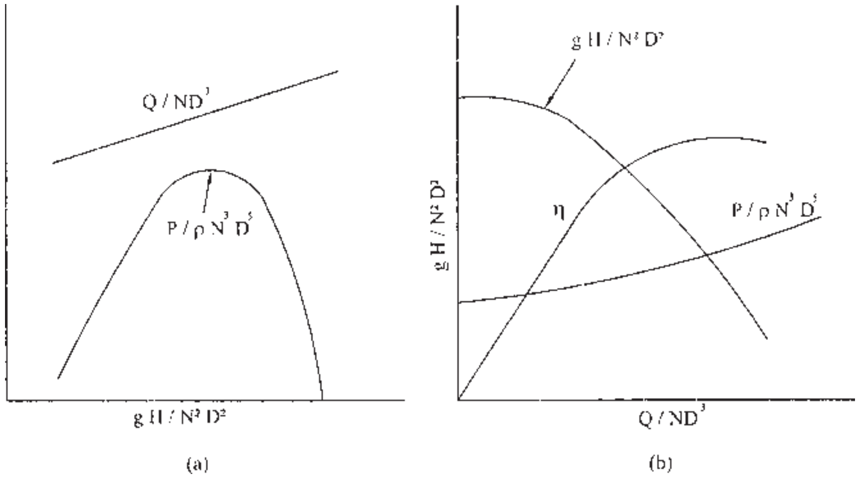

Figure 1.7 Performance characteristics of hydraulic machines: (a) hydraulic turbine, (b) hydraulic pump.

Typical dimensionless characteristic curves for a hydraulic turbine and pump are shown in Fig. 1.7 (a) and (b), respectively. These characteristic curves are also the curves of any other combination of P, N, Q, and H for a given machine or for any other geometrically similar machine.

1.8 MODEL TESTING

Some very large hydraulic machines are tested in a model form before making the full-sized machine. After the result is obtained from the model, one may transpose the results from the model to the full-sized machine. Therefore if the curves shown in Fig 1.7 have been obtained for a completely similar model, these same curves would apply to the full-sized prototype machine.

1.9 GEOMETRIC SIMILARITY

For geometric similarity to exist between the model and prototype, both of them should be identical in shape but differ only in size. Or, in other words, for geometric similarity between the model and the prototype, the ratios of all the corresponding linear dimensions should be equal.

Let L_p be the length of the prototype, B_p, the breadth of the prototype, D_p, the depth of the prototype, and L_m, B_m, and D_m the corresponding dimensions of the model. For geometric similarity, linear ratio (or scale ratio) is given by

$$L_r = \frac{L_p}{L_m} = \frac{B_p}{B_m} = \frac{D_p}{D_m} \tag{1.19}$$

Similarly, the area ratio between prototype and model is given by

$$A_r = \left(\frac{L_p}{L_m}\right)^2 = \left(\frac{B_p}{B_m}\right)^2 = \left(\frac{D_p}{D_m}\right)^2 \tag{1.20}$$

and the volume ratio

$$V_r = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{B_p}{B_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3 \tag{1.21}$$

1.10 KINEMATIC SIMILARITY

For kinematic similarity, both model and prototype have identical motions or velocities. If the ratio of the corresponding points is equal, then the velocity ratio of the prototype to the model is

$$V_r = \frac{V_1}{v_1} = \frac{V_2}{v_2} \tag{1.22}$$

where V_1 is the velocity of liquid in the prototype at point 1, V_2, the velocity of liquid in the prototype at point 2, v_1, the velocity of liquid in the model at point 1, and v_2 is the velocity of liquid in the model at point 2.

1.11 DYNAMIC SIMILARITY

If model and prototype have identical forces acting on them, then dynamic similarity will exist. Let F_1 be the forces acting on the prototype at point 1, and F_2 be the forces acting on the prototype at point 2. Then the force ratio to establish dynamic similarity between the prototype and the model is given by

$$F_r = \frac{F_{p1}}{F_{m1}} = \frac{F_{p2}}{F_{m2}} \tag{1.23}$$

1.12 PROTOTYPE AND MODEL EFFICIENCY

Let us suppose that the similarity laws are satisfied, η_p and η_m are the prototype and model efficiencies, respectively. Now from similarity laws, representing the model and prototype by subscripts m and p respectively,

$$\frac{H_p}{(N_p D_p)^2} = \frac{H_m}{(N_m D_m)^2} \quad \text{or} \quad \frac{H_p}{H_m} = \left(\frac{N_p}{N_m}\right)^2 \left(\frac{D_p}{D_m}\right)^2$$

$$\frac{Q_p}{N_p D_p^3} = \frac{Q_m}{N_m D_m^3} \quad \text{or} \quad \frac{Q_p}{Q_m} = \left(\frac{N_p}{N_m}\right)\left(\frac{D_p}{D_m}\right)^3$$

$$\frac{P_p}{N_p^3 D_p^5} = \frac{P_m}{N_m^3 D_m^5} \quad \text{or} \quad \frac{P_p}{P_m} = \left(\frac{N_p}{N_m}\right)^3\left(\frac{D_p}{D_m}\right)^5$$

Turbine efficiency is given by

$$\eta_t = \frac{\text{Power transferred from fluid}}{\text{Fluid power available.}} = \frac{P}{\rho g Q H}$$

Hence, $\dfrac{\eta_m}{\eta_p} = \left(\dfrac{P_m}{P_p}\right)\left(\dfrac{Q_p}{Q_m}\right)\left(\dfrac{H_p}{H_m}\right) = 1.$

Thus, the efficiencies of the model and prototype are the same providing the similarity laws are satisfied.

1.13 PROPERTIES INVOLVING THE MASS OR WEIGHT OF THE FLUID

1.13.1 Specific Weight (γ)

The weight per unit volume is defined as specific weight and it is given the symbol γ (gamma). For the purpose of all calculations relating to hydraulics, fluid machines, the specific weight of water is taken as 1000 l/m^3. In S.I. units, the specific weight of water is taken as 9.80 kN/m^3.

1.13.2 Mass Density (ρ)

The mass per unit volume is mass density. In S.I. systems, the units are kilograms per cubic meter or NS^2/m^4. Mass density, often simply called density, is given the greek symbol ρ (rho). The mass density of water at 15.5° is 1000 kg/m^3.

1.13.3 Specific Gravity (sp.gr.)

The ratio of the specific weight of a given liquid to the specific weight of water at a standard reference temperature is defined as specific gravity. The standard reference temperature for water is often taken as 4°C Because specific gravity is a ratio of specific weights, it is dimensionless and, of course, independent of system of units used.

1.13.4 Viscosity (μ)

We define viscosity as the property of a fluid, which offers resistance to the relative motion of fluid molecules. The energy loss due to friction in a flowing fluid is due to the viscosity. When a fluid moves, a shearing stress develops in it. The magnitude of the shearing stress depends on the viscosity of the fluid. Shearing stress, denoted by the symbol τ (tau) can be defined as the force required to slide on unit area layers of a substance over another. Thus τ is a force divided by an area and can be measured in units N/m^2 or Pa. In a fluid such as water, oil, alcohol, or other common liquids, we find that the magnitude of the shearing stress is directly proportional to the change of velocity between different positions in the fluid. This fact can be stated mathematically as

$$\tau = \mu\left(\frac{\Delta v}{\Delta y}\right) \tag{1.24}$$

where $\frac{\Delta v}{\Delta y}$ is the velocity gradient and the constant of proportionality μ is called the dynamic viscosity of fluid.

Units for Dynamic Viscosity

Solving for μ gives

$$\mu = \frac{\tau}{\Delta v/\Delta y} = \tau\left(\frac{\Delta y}{\Delta v}\right)$$

Substituting the units only into this equation gives

$$\mu = \frac{\mathrm{N}}{\mathrm{m}^2} \times \frac{\mathrm{m}}{\mathrm{m/s}} = \frac{\mathrm{N} \times \mathrm{s}}{\mathrm{m}^2}$$

Since Pa is a shorter symbol representing N/m^2, we can also express μ as

$$\mu = \mathrm{Pa} \cdot \mathrm{s}$$

1.13.5 Kinematic Viscosity (ν)

The ratio of the dynamic viscosity to the density of the fluid is called the kinematic viscosity υ (nu). It is defined as

$$\nu = \frac{\mu}{\rho} = \mu(1/\rho) = \frac{\mathrm{kg}}{\mathrm{ms}} \times \frac{\mathrm{m}^3}{\mathrm{kg}} = \frac{\mathrm{m}^2}{\mathrm{s}} \tag{1.25}$$

Any fluid that behaves in accordance with Eq. (1.25) is called a Newtonian fluid.

1.14 COMPRESSIBLE FLOW MACHINES

Compressible fluids are working substances in gas turbines, centrifugal and axial flow compressors. To include the compressibility of these types of fluids (gases), some new variables must be added to those already discussed in the case of hydraulic machines and changes must be made in some of the definitions used. The important parameters in compressible flow machines are pressure and temperature.

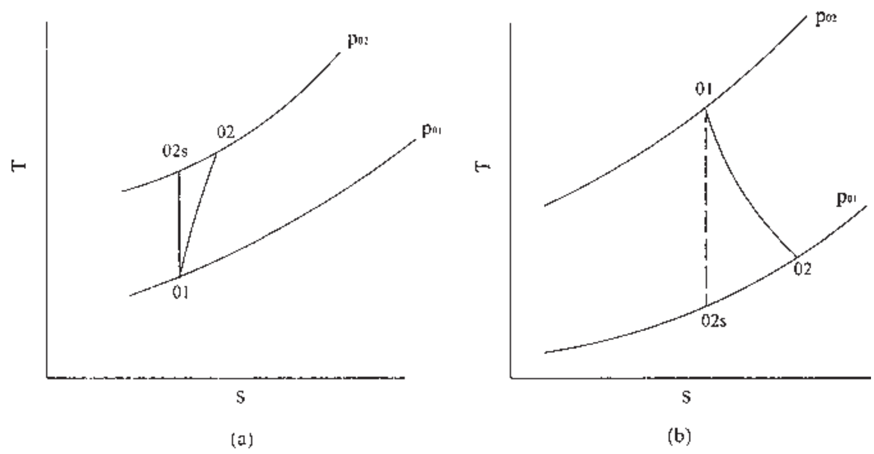

Figure 1.8 Compression and expansion in compressible flow machines: (a) compression, (b) expansion.

In Fig. 1.8 T-s charts for compression and expansion processes are shown.

Isentropic compression and expansion processes are represented by s and the subscript 0 refers to stagnation or total conditions. 1 and 2 refer to the inlet and outlet states of the gas, respectively. The pressure at the outlet, P_{02}, can be expressed as follows

$$P_{02} = f\left(D, N, m, P_{01}, T_{01}, T_{02}, \rho_{01}, \rho_{02}, \mu\right) \tag{1.26}$$

The pressure ratio P_{02}/P_{01} replaces the head H, while the mass flow rate m (kg/s) replaces Q. Using the perfect gas equation, density may be written as $\rho = P/RT$. Now, deleting density and combining R with T, the functional relationship can be written as

$$P_{02} = f(P_{01}, RT_{01}, RT_{02}, m, N, D, \mu) \tag{1.27}$$

Substituting the basic dimensions and equating the indices, the following fundamental relationship may be obtained

$$\frac{P_{02}}{P_{01}} = f\left(\left(\frac{RT_{02}}{RT_{01}}\right), \left(\frac{\left(\frac{m}{RT_{01}}\right)^{1/2}}{P_{01}D^2}\right), \left(\frac{ND}{(RT_{01})^{1/2}}\right), \mathrm{Re}\right) \tag{1.28}$$

In Eq. (1.28), R is constant and may be eliminated. The Reynolds number in most cases is very high and the flow is turbulent and therefore changes in this parameter over the usual operating range may be neglected. However, due to

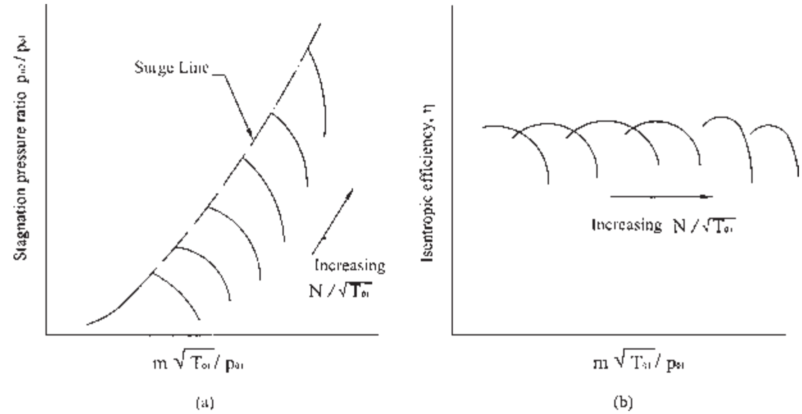

Figure 1.9 Axial flow compressor characteristics: (a) pressure ratio, (b) efficiency.

large changes of density, a significant reduction in *Re* can occur which must be taken into consideration. For a constant diameter machine, the diameter D may be ignored, and hence Eq. (1.28) becomes

$$\frac{P_{02}}{P_{01}} = f\left(\left(\frac{T_{02}}{T_{01}}\right), \left(\frac{mT_{01}^{1/2}}{P_{01}}\right), \left(\frac{N}{T_{01}^{1/2}}\right)\right) \tag{1.29}$$

In Eq. (1.29) some of the terms are new and no longer dimensionless. For a particular machine, it is typical to plot P_{02}/P_{01} and T_{02}/T_{01} against the mass flow

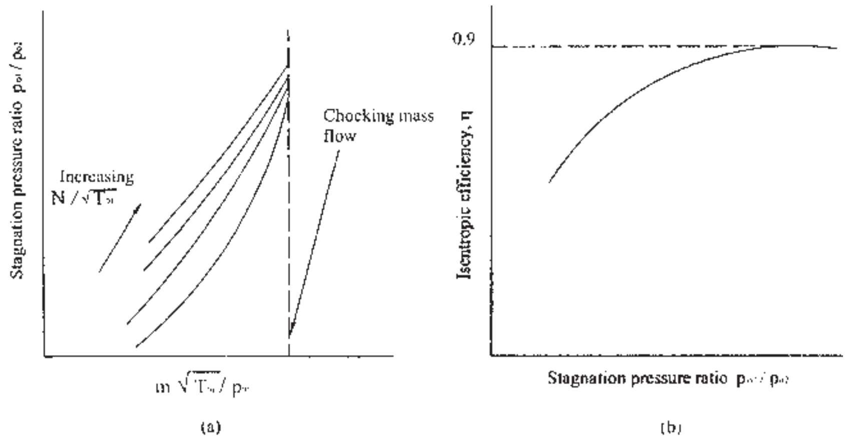

Figure 1.10 Axial flow gas turbine characteristics: (a) pressure ratio, (b) efficiency.

rate parameter $mT_{01}^{1/2}/P_{01}$ for different values of the speed parameter $N/T_{01}^{1/2}$. Equation (1.28) must be used if it is required to change the size of the machine. The term $ND/(RT_{01})^{1/2}$ indicates the Mach number effect. This occurs because the impeller velocity $v \propto ND$ and the acoustic velocity $a_{01} \propto RT_{01}$, while the Mach number

$$M = V/a_{01} \tag{1.30}$$

The performance curves for an axial flow compressor and turbine are shown in Figs. 1.9 and 1.10.

1.15 BASIC THERMODYNAMICS, FLUID MECHANICS, AND DEFINITIONS OF EFFICIENCY

In this section, the basic physical laws of fluid mechanics and thermodynamics will be discussed. These laws are:

1. The continuity equation.
2. The First Law of Thermodynamics.
3. Newton's Second Law of Motion.
4. The Second Law of Thermodynamics.

The above items are comprehensively dealt with in books on thermodynamics with engineering applications, so that much of the elementary discussion and analysis of these laws need not be repeated here.

1.16 CONTINUITY EQUATION

For steady flow through a turbomachine, m remains constant. If A_1 and A_2 are the flow areas at Secs. 1 and 2 along a passage respectively, then

$$\dot{m} = \rho_1 A_1 C_1 = \rho_2 A_2 C_2 = \text{constant} \tag{1.31}$$

where ρ_1, is the density at section 1, ρ_2, the density at section 2, C_1, the velocity at section 1, and C_2, is the velocity at section 2.

1.17 THE FIRST LAW OF THERMODYNAMICS

According to the First Law of Thermodynamics, if a system is taken through a complete cycle during which heat is supplied and work is done, then

$$\oint (\delta Q - \delta W) = 0 \tag{1.32}$$

where $\oint \delta Q$ represents the heat supplied to the system during this cycle and $\oint \delta W$

the work done by the system during the cycle. The units of heat and work are taken to be the same. During a change of state from 1 to 2, there is a change in the internal energy of the system

$$U_2 - U_1 = \int_1^2 (\delta Q - \delta W) \tag{1.33}$$

For an infinitesimal change of state

$$dU = \delta Q - \delta W \tag{1.34}$$

1.17.1 The Steady Flow Energy Equation

The First Law of Thermodynamics can be applied to a system to find the change in the energy of the system when it undergoes a change of state. The total energy of a system, E may be written as:

$$E = \text{Internal Energy} + \text{Kinetic Energy} + \text{Potential Energy}$$

$$E = U + K.E. + P.E. \tag{1.35}$$

where U is the internal energy. Since the terms comprising E are point functions, we can write Eq. (1.35) in the following form

$$dE = dU + d(\text{K.E.}) + d(\text{P.E.}) \tag{1.36}$$

The First Law of Thermodynamics for a change of state of a system may therefore be written as follows

$$\delta Q = dU + d(\text{KE}) + d(\text{PE}) + \delta W \tag{1.37}$$

Let subscript 1 represents the system in its initial state and 2 represents the system in its final state, the energy equation at the inlet and outlet of any device may be written

$$Q_{1-2} = U_2 - U_1 + \frac{m(C_2^2 - C_1^2)}{2} + mg(Z_2 - Z_1) + W_{1-2} \tag{1.38}$$

Equation (1.38) indicates that there are differences between, or changes in, similar forms of energy entering or leaving the unit. In many applications, these differences are insignificant and can be ignored. Most closed systems encountered in practice are stationary; i.e. they do not involve any changes in their velocity or the elevation of their centers of gravity during a process. Thus, for stationary closed systems, the changes in kinetic and potential energies are negligible (i.e. $\Delta(\text{K.E.}) = \Delta(\text{P.E.}) = 0$), and the first law relation reduces to

$$Q - W = \Delta E \tag{1.39}$$

If the initial and final states are specified the internal energies 1 and 2 can easily be determined from property tables or some thermodynamic relations.

1.17.2 Other Forms of the First Law Relation

The first law can be written in various forms. For example, the first law relation on a unit-mass basis is

$$q - w = \Delta e(\mathrm{kJ/kg}) \tag{1.40}$$

Dividing Eq. (1.39) by the time interval Δt and taking the limit as $\Delta t \rightarrow 0$ yields the rate form of the first law

$$\dot{Q} - \dot{W} = \frac{\mathrm{d}E}{\mathrm{d}t} \tag{1.41}$$

where $\dot{Q}$ is the rate of net heat transfer, $\dot{W}$ the power, and $\frac{\mathrm{d}E}{\mathrm{d}t}$ is the rate of change of total energy. Equations. (1.40) and (1.41) can be expressed in differential form

$$\delta Q - \delta W = \mathrm{d}E(\mathrm{kJ}) \tag{1.42}$$

$$\delta q - \delta w = \mathrm{d}e(\mathrm{kJ/kg}) \tag{1.43}$$

For a cyclic process, the initial and final states are identical; therefore, $\Delta E = E_2 - E_1$.

Then the first law relation for a cycle simplifies to

$$Q - W = 0(\mathrm{kJ}) \tag{1.44}$$

That is, the net heat transfer and the net work done during a cycle must be equal. Defining the stagnation enthalpy by: $h_0 = h + \frac{1}{2}c^2$ and assuming $g\ (Z_2 - Z_1)$ is negligible, the steady flow energy equation becomes

$$\dot{Q} - \dot{W} = \dot{m}(h_{02} - h_{01}) \tag{1.45}$$

Most turbomachinery flow processes are adiabatic, and so $\dot{Q} = 0$. For work producing machines, $\dot{W} > 0$; so that

$$\dot{W} = \dot{m}(h_{01} - h_{02}) \tag{1.46}$$

For work absorbing machines (compressors) $W < 0$; so that

$$\dot{W} \rightarrow -\dot{W} = \dot{m}(h_{02} - h_{01}) \tag{1.47}$$

1.18 NEWTON'S SECOND LAW OF MOTION

Newton's Second Law states that the sum of all the forces acting on a control volume in a particular direction is equal to the rate of change of momentum of the fluid across the control volume. For a control volume with fluid entering with uniform velocity C_1 and leaving with uniform velocity C_2, then

$$\sum F = \dot{m}(C_2 - C_1) \tag{1.48}$$

Equation (1.48) is the one-dimensional form of the steady flow momentum equation, and applies for linear momentum. However, turbomachines have impellers that rotate, and the power output is expressed as the product of torque and angular velocity. Therefore, angular momentum is the most descriptive parameter for this system.

1.19 THE SECOND LAW OF THERMODYNAMICS: ENTROPY

This law states that for a fluid passing through a cycle involving heat exchanges

$$\oint \frac{\delta Q}{T} \leq 0 \tag{1.49}$$

where δQ is an element of heat transferred to the system at an absolute temperature T. If all the processes in the cycle are reversible, so that $\delta Q = \delta Q_R$, then

$$\oint \frac{\delta Q_R}{T} = 0 \tag{1.50}$$

The property called entropy, for a finite change of state, is then given by

$$S_2 - S_1 = \int_1^2 \frac{\delta Q_R}{T} \tag{1.51}$$

For an incremental change of state

$$dS = m\,ds = \frac{\delta Q_R}{T} \tag{1.52}$$

where m is the mass of the fluid. For steady flow through a control volume in which the fluid experiences a change of state from inlet 1 to outlet 2,

$$\int_1^2 \frac{\delta \dot{Q}}{T} \leq \dot{m}(s_2 - s_1) \tag{1.53}$$

For adiabatic process, $\delta Q = 0$ so that

$$s_2 \geq s_1 \tag{1.54}$$

For reversible process

$$s_2 = s_1 \tag{1.55}$$

In the absence of motion, gravity and other effects, the first law of thermodynamics, Eq. (1.34) becomes

$$T\mathrm{d}s = \mathrm{d}u + p\mathrm{d}v \tag{1.56}$$

Putting $h = u + pv$ and $\mathrm{d}h = \mathrm{d}u + p\mathrm{d}v + v\mathrm{d}p$ in Eq. (1.56) gives

$$T\mathrm{d}s = \mathrm{d}h - v\mathrm{d}p \tag{1.57}$$

1.20 EFFICIENCY AND LOSSES

Let H be the head parameter (m), Q discharge (m^3/s)

The waterpower supplied to the machine is given by

$$P = \rho Q g H (\text{in watts}) \tag{1.58}$$

and letting $\rho = 1000\,\mathrm{kg/m^3}$,

$$= QgH(\text{in kW})$$

Now, let ΔQ be the amount of water leaking from the tail race. This is the amount of water, which is not providing useful work.

Then:

$$\text{Power wasted} = \Delta Q(gH)(\mathrm{kW})$$

For volumetric efficiency, we have

$$\eta_\nu = \frac{Q - \Delta Q}{Q} \tag{1.59}$$

Net power supplied to turbine

$$= (Q - \Delta Q)gH(\mathrm{kW}) \tag{1.60}$$

If H_r is the runner head, then the hydraulic power generated by the runner is given by

$$P_\mathrm{h} = (Q - \Delta Q)gH_\mathrm{r}(\mathrm{kW}) \tag{1.61}$$

The hydraulic efficiency, η_h is given by

$$\eta_\mathrm{h} = \frac{\text{Hydraulic output power}}{\text{Hydraulic input power}} = \frac{(Q - \Delta Q)gH_\mathrm{r}}{(Q - \Delta Q)gH} = \frac{H_\mathrm{r}}{H} \tag{1.62}$$

If P_m represents the power loss due to mechanical friction at the bearing, then the available shaft power is given by

$$P_s = P_h - P_m \tag{1.63}$$

Mechanical efficiency is given by

$$\eta_m = \frac{P_s}{P_h} = \frac{P_s}{P_m - P_s} \tag{1.64}$$

The combined effect of all these losses may be expressed in the form of overall efficiency. Thus

$$\eta_0 = \frac{P_s}{WP} = \eta_m \frac{P_h}{WP}$$

$$= \eta_m \frac{WP(Q - \Delta Q)}{WPQ\Delta H} = \eta_m \eta_v \eta_h \tag{1.65}$$

1.21 STEAM AND GAS TURBINES

Figure 1.11 shows an enthalpy–entropy or Mollier diagram. The process is represented by line 1–2 and shows the expansion from pressure P_1 to a lower pressure P_2. The line 1–2s represents isentropic expansion. The actual

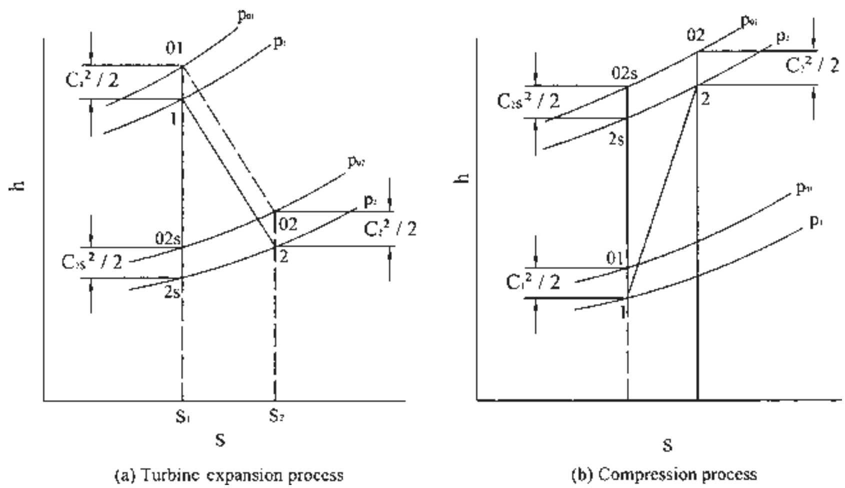

Figure 1.11 Enthalpy–entropy diagrams for turbines and compressors: (a) turbine expansion process, (b) compression process.

turbine-specific work is given by

$$W_t = h_{01} - h_{02} = (h_1 - h_2) + \frac{1}{2}(C_1^2 - C_2^2) \tag{1.66}$$

Similarly, the isentropic turbine rotor specific work between the same two pressures is

$$W_t' = h_{01} - h_{02s} = (h_1 - h_{2s}) + \frac{1}{2}\left(C_1^2 - C_{2s}^2\right) \tag{1.67}$$

Efficiency can be expressed in several ways. The choice of definitions depends largely upon whether the kinetic energy at the exit is usefully utilized or wasted. In multistage gas turbines, the kinetic energy leaving one stage is utilized in the next stage. Similarly, in turbojet engines, the energy in the gas exhausting through the nozzle is used for propulsion. For the above two cases, the turbine isentropic efficiency η_{tt} is defined as

$$\eta_{tt} = \frac{W_t}{W_t'} = \frac{h_{01} - h_{02}}{h_{01} - h_{02s}} \tag{1.68}$$

When the exhaust kinetic energy is not totally used but not totally wasted either, the total-to-static efficiency, η_{ts}, is used. In this case, the ideal or isentropic turbine work is that obtained between static points 01 and 2s. Thus

$$\eta_{ts} = \frac{h_{01} - h_{02}}{h_{01} - h_{02s} + \frac{1}{2}C_{2s}^2} = \frac{h_{01} - h_{02}}{h_{01} - h_{2s}} \tag{1.69}$$

If the difference between inlet and outlet kinetic energies is small, Eq. (1.69) becomes

$$\eta_{ts} = \frac{h_1 - h_2}{h_1 - h_{2s} + \frac{1}{2}C_{1s}^2}$$

An example where the outlet kinetic energy is wasted is a turbine exhausting directly to the atmosphere rather than exiting through a diffuser.

1.22 EFFICIENCY OF COMPRESSORS

The isentropic efficiency of the compressor is defined as

$$\eta_c = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} \tag{1.70}$$

If the difference between inlet and outlet kinetic energies is small, $\frac{1}{2}C_1^2 = \frac{1}{2}C_2^2$ and

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \tag{1.71}$$

1.23 POLYTROPIC OR SMALL-STAGE EFFICIENCY

Isentropic efficiency as described above can be misleading if used for compression and expansion processes in several stages. Turbomachines may be used in large numbers of very small stages irrespective of the actual number of stages in the machine. If each small stage has the same efficiency, then the isentropic efficiency of the whole machine will be different from the small stage efficiency, and this difference is dependent upon the pressure ratio of the machine.

Isentropic efficiency of compressors tends to decrease and isentropic efficiency of turbines tends to increase as the pressure ratios for which the machines are designed are increased. This is made more apparent in the following argument.

Consider an axial flow compressor, which is made up of several stages, each stage having equal values of η_c, as shown in Fig. 1.12.

Then the overall temperature rise can be expressed by

$$\Delta T = \sum \frac{\Delta T_s'}{\eta_s} = \frac{1}{\eta_s} \sum \Delta T_s'$$

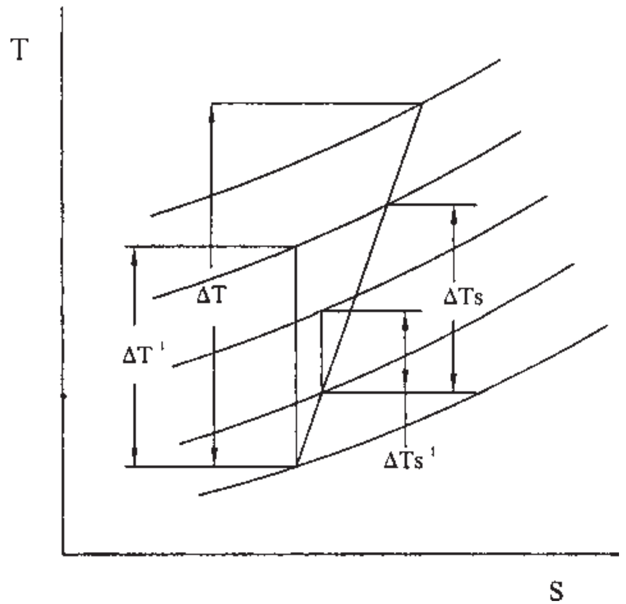

Figure 1.12 Compression process in stages.

(Prime symbol is used for isentropic temperature rise, and subscript s is for stage temperature).

Also, $\Delta T = \Delta^{T'}/\eta_c$ by definition of η_c, and thus: $\eta_s/\eta_c = \sum \Delta T_s{}'/\Delta T'$. It is clear from Fig. 1.12 that $\sum \Delta T'_s > \Delta T'$. Hence, $\eta_c < \eta_s$ and the difference will increase with increasing pressure ratio. The opposite effect is obtained in a turbine where η_s (i.e., small stage efficiency) is less than the overall efficiency of the turbine.

The above discussions have led to the concept of polytropic efficiency, η_∞, which is defined as the isentropic efficiency of an elemental stage in the process such that it is constant throughout the entire process.

The relationship between a polytropic efficiency, which is constant through the compressor, and the overall efficiency η_c may be obtained for a gas of constant specific heat.

For compression,

$$\eta_{\infty c} = \frac{dT'}{dT} = \text{constant}$$

But, $\frac{T}{p^{(\gamma-1)/\gamma}} =$ constant for an isentropic process, which in differential form is

$$\frac{dT'}{dT} = \frac{\gamma - 1}{\gamma}\frac{dP}{P}$$

Now, substituting dT' from the previous equation, we have

$$\eta_{\infty c}\frac{dT'}{dT} = \frac{\gamma - 1}{\gamma}\frac{dP}{P}$$

Integrating the above equation between the inlet 1 and outlet 2, we get

$$\eta_{\infty c} = \frac{\ln(P_2/P_1)^{\frac{\gamma-1}{\gamma}}}{\ln(T_2/T_1)} \tag{1.72}$$

Equation (1.72) can also be written in the form

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma\eta_{\infty c}}} \tag{1.73}$$

The relation between $\eta_{\infty c}$ and η_c is given by

$$\eta_c = \frac{(T_2'/T_1) - 1}{(T_2/T_1) - 1} = \frac{(P_2/P_1)^{\frac{\gamma-1}{\gamma}} - 1}{(P_2/P_1)^{\frac{\gamma-1}{\gamma\eta_{\infty c}}} - 1} \tag{1.74}$$

From Eq. (1.74), if we write $\frac{\gamma-1}{\gamma\eta_{\infty c}}$ as $\frac{n-1}{n}$, Eq. (1.73) is the functional relation between P and T for a polytropic process, and thus it is clear that the non isentropic process is polytropic.

Similarly, for an isentropic expansion and polytropic expansion, the following relations can be developed between the inlet 1 and outlet 2:

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\eta_{\infty t}(\gamma-1)}{\gamma}}$$

and

$$\eta_t = \frac{1 - \left(\frac{1}{P_1/P_2}\right)^{\frac{\eta_{\infty t}(\gamma-1)}{\gamma}}}{1 - \left(\frac{1}{P_1/P_2}\right)^{\frac{(\gamma-1)}{\gamma}}} \tag{1.75}$$

where $\eta_{\infty t}$ is the small-stage or polytropic efficiency for the turbine.

Figure 1.13 shows the overall efficiency related to the polytropic efficiency for a constant value of $\gamma = 1.4$, for varying polytropic efficiencies and for varying pressure ratios.

As mentioned earlier, the isentropic efficiency for an expansion process exceeds the small-stage efficiency. Overall isentropic efficiencies have been

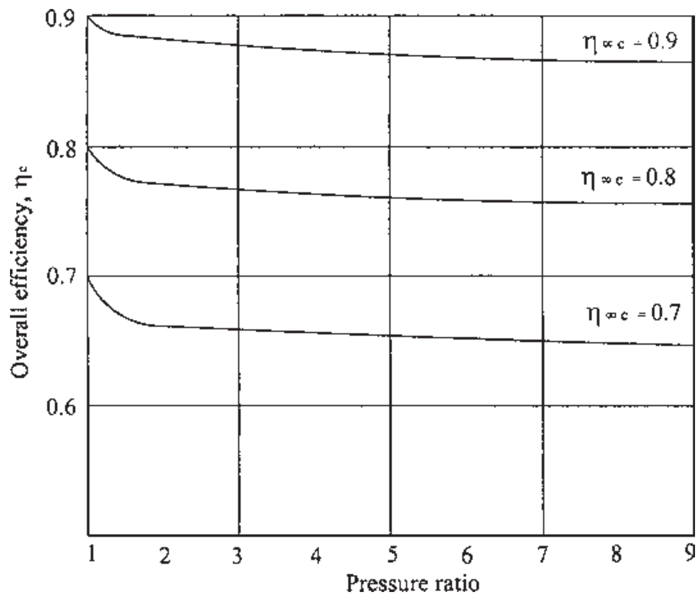

Figure 1.13 Relationships among overall efficiency, polytropic efficiency, and pressure ratio.

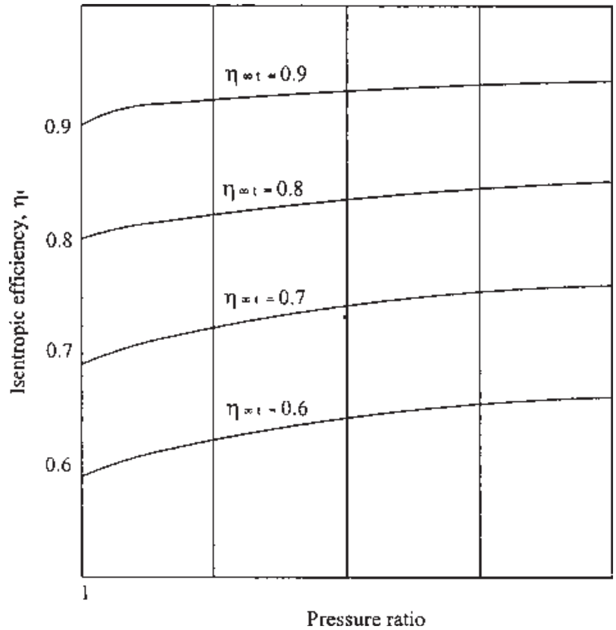

Figure 1.14 Turbine isentropic efficiency against pressure ratio for various polytropic efficiencies ($\gamma = 1.4$).

calculated for a range of pressure ratios and different polytropic efficiencies. These relationships are shown in Fig. 1.14.

1.24 NOZZLE EFFICIENCY

The function of the nozzle is to transform the high-pressure temperature energy (enthalpy) of the gasses at the inlet position into kinetic energy. This is achieved by decreasing the pressure and temperature of the gasses in the nozzle.

From Fig. 1.15, it is clear that the maximum amount of transformation will result when we have an isentropic process between the pressures at the entrance and exit of the nozzle. Such a process is illustrated as the path 1–2s. Now, when nozzle flow is accompanied by friction, the entropy will increase. As a result, the path is curved as illustrated by line 1–2. The difference in the enthalpy change between the actual process and the ideal process is due to friction. This ratio is known as the nozzle adiabatic efficiency and is called nozzle efficiency (η_n) or jet

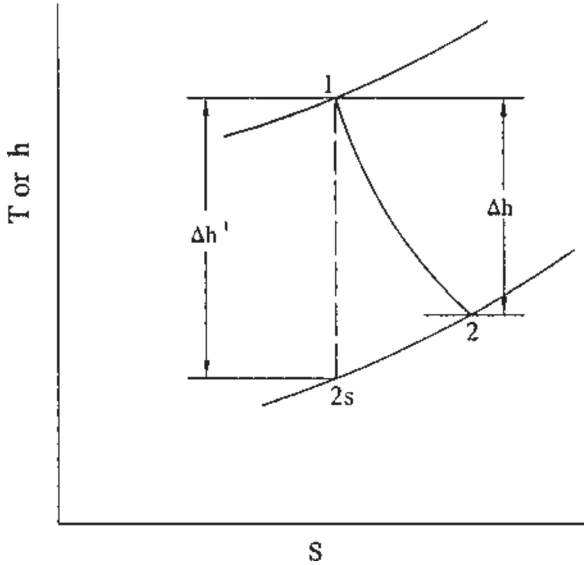

Figure 1.15 Comparison of ideal and actual nozzle expansion on a T-s or h–s plane.

pipe efficiency (η_j). This efficiency is given by:

$$\eta_j = \frac{\Delta h}{\Delta^{h\prime}} = \frac{h_{01} - h_{02}}{h_{01} - h_{02}{}'} = \frac{c_p(T_{01} - T_{02})}{c_p(T_{01} - T_{02}{}')} \tag{1.76}$$

1.25 DIFFUSER EFFICIENCY

The diffuser efficiency η_d is defined in a similar manner to compressor efficiency (see Fig. 1.16):

$$\begin{aligned}\eta_d &= \frac{\text{Isentropic enthalpy rise}}{\text{Actual enthalpy rise}} \\ &= \frac{h_{2s} - h_1}{h_2 - h_1}\end{aligned} \tag{1.77}$$

The purpose of diffusion or deceleration is to convert the maximum possible kinetic energy into pressure energy. The diffusion is difficult to achieve and is rightly regarded as one of the main problems of turbomachinery design. This problem is due to the growth of boundary layers and the separation of the fluid molecules from the diverging part of the diffuser. If the rate of diffusion is too rapid, large losses in stagnation pressure are inevitable. On the other hand, if

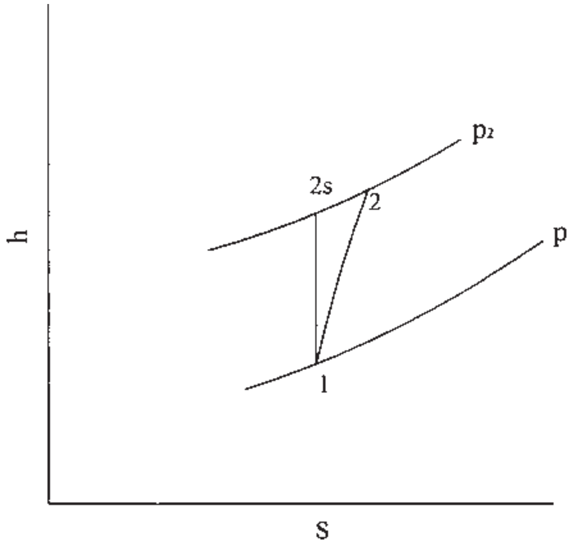

Figure 1.16 Mollier diagram for the diffusion process.

the rate of diffusion is very low, the fluid is exposed to an excessive length of wall and friction losses become predominant. To minimize these two effects, there must be an optimum rate of diffusion.

1.26 ENERGY TRANSFER IN TURBOMACHINERY

This section deals with the kinematics and dynamics of turbomachines by means of definitions, diagrams, and dimensionless parameters. The kinematics and dynamic factors depend on the velocities of fluid flow in the machine as well as the rotor velocity itself and the forces of interaction due to velocity changes.

1.27 THE EULER TURBINE EQUATION

The fluid flows through the turbomachine rotor are assumed to be steady over a long period of time. Turbulence and other losses may then be neglected, and the mass flow rate m is constant. As shown in Fig. 1.17, let ω (omega) be the angular velocity about the axis A–A.

Fluid enters the rotor at point 1 and leaves at point 2.

In turbomachine flow analysis, the most important variable is the fluid velocity and its variation in the different coordinate directions. In the designing of blade shapes, velocity vector diagrams are very useful. The flow in and across

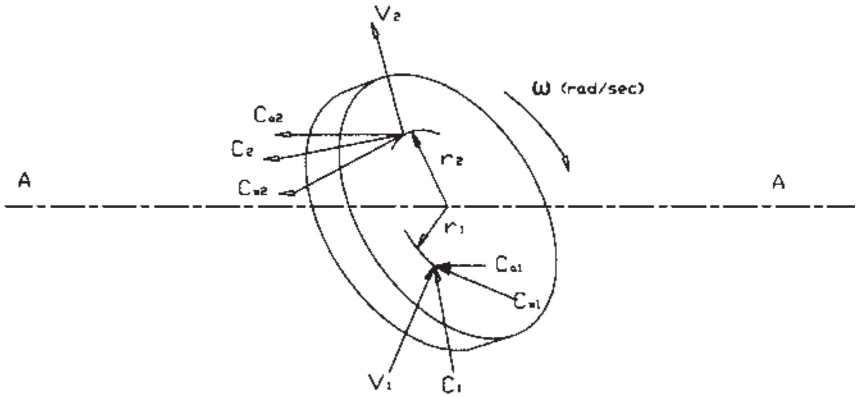

Figure 1.17 Velocity components for a generalized rotor.

the stators, the absolute velocities are of interest (i.e., C). The flow velocities across the rotor relative to the rotating blade must be considered. The fluid enters with velocity C_1, which is at a radial distance r_1 from the axis A–A. At point 2 the fluid leaves with absolute velocity (that velocity relative to an outside observer). The point 2 is at a radial distance r_2 from the axis A–A. The rotating disc may be either a turbine or a compressor. It is necessary to restrict the flow to a steady flow, i.e., the mass flow rate is constant (no accumulation of fluid in the rotor). The velocity C_1 at the inlet to the rotor can be resolved into three components; viz.;

C_{a1} — Axial velocity in a direction parallel to the axis of the rotating shaft.

C_{r1} — Radial velocity in the direction normal to the axis of the rotating shaft.

C_{w1} — whirl or tangential velocity in the direction normal to a radius.

Similarly, exit velocity C_2 can be resolved into three components; that is, C_{a2}, C_{r2}, and C_{w2}. The change in magnitude of the axial velocity components through the rotor gives rise to an axial force, which must be taken by a thrust bearing to the stationary rotor casing. The change in magnitude of the radial velocity components produces radial force. Neither has any effect on the angular motion of the rotor. The whirl or tangential components C_w produce the rotational effect. This may be expressed in general as follows:

The unit mass of fluid entering at section 1 and leaving in any unit of time produces:

The angular momentum at the inlet: $C_{w1}r_1$

The angular momentum at the outlet: $C_{w2}r_2$

And therefore the rate of change of angular momentum $= C_{w1}r_1 - C_{w2}r_2$

By Newton's laws of motion, this is equal to the summation of all the applied forces on the rotor; i.e., the net torque of the rotor τ (tau). Under steady flow conditions, using mass flow rate m, the torque exerted by or acting on the rotor will be:

$$\tau = m(C_{w1}r_1 - C_{w2}r_2)$$

Therefore the rate of energy transfer, W, is the product of the torque and the angular velocity of the rotor ω (omega), so:

$$W = \tau\omega = m\omega(C_{w1}r_1 - C_{w2}r_2)$$

For unit mass flow, energy will be given by:

$$W = \omega(C_{w1}r_1 - C_{w2}r_2) = (C_{w1}r_1\omega - C_{w2}r_2\omega)$$

But, $\omega\, r_1 = U_1$ and $\omega\, r_2 = U_2$.

$$\text{Hence, } W = (C_{w1}U_1 - C_{w2}U_2), \tag{1.78}$$

where, W is the energy transferred per unit mass, and U_1 and U_2 are the rotor speeds at the inlet and the exit respectively. Equation (1.78) is referred to as Euler's turbine equation. The standard thermodynamic sign convention is that work done by a fluid is positive, and work done on a fluid is negative. This means the work produced by the turbine is positive and the work absorbed by the compressors and pumps is negative. Therefore, the energy transfer equations can be written separately as

$$W = (C_{w1}U_1 - C_{w2}U_2) \text{ for turbine}$$

and

$$W = (C_{w2}U_2 - C_{w1}U_1) \text{ for compressor and pump.}$$

The Euler turbine equation is very useful for evaluating the flow of fluids that have very small viscosities, like water, steam, air, and combustion products.

To calculate torque from the Euler turbine equation, it is necessary to know the velocity components C_{w1}, C_{w2}, and the rotor speeds U_1 and U_2 or the velocities V_1, V_2, C_{r1}, C_{r2} as well as U_1 and U_2. These quantities can be determined easily by drawing the velocity triangles at the rotor inlet and outlet, as shown in Fig. 1.18. The velocity triangles are key to the analysis of turbo-machinery problems, and are usually combined into one diagram. These triangles are usually drawn as a vector triangle:

Since these are vector triangles, the two velocities U and V are relative to one another, so that the tail of V is at the head of U. Thus the vector sum of U and V is equal to the vector C. The flow through a turbomachine rotor, the absolute velocities C_1 and C_2 as well as the relative velocities V_1 and V_2 can have three

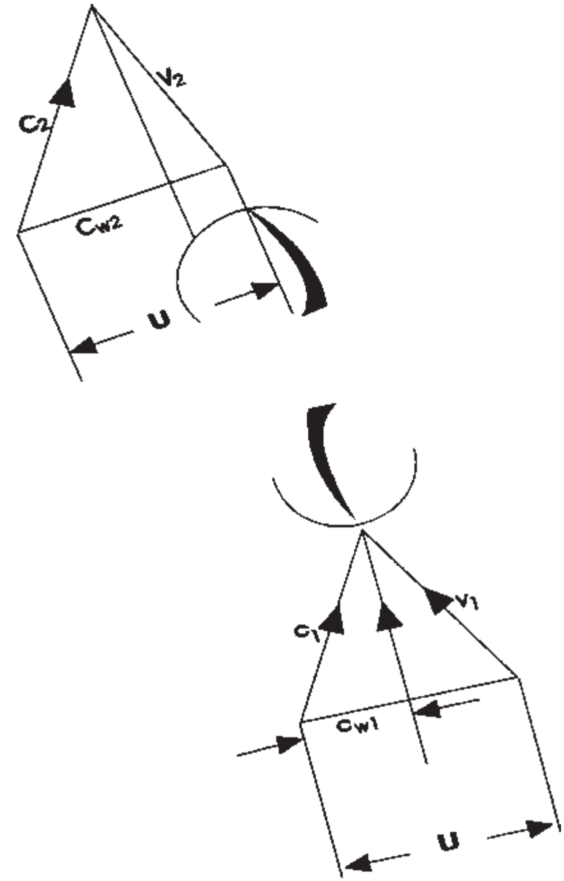

Figure 1.18 Velocity triangles for a rotor.

components as mentioned earlier. However, the two velocity components, one tangential to the rotor (C_w) and another perpendicular to it are sufficient. The component C_r is called the meridional component, which passes through the point under consideration and the turbomachine axis. The velocity components C_{r1} and C_{r2} are the flow velocity components, which may be axial or radial depending on the type of machine.

1.28 COMPONENTS OF ENERGY TRANSFER

The Euler equation is useful because it can be transformed into other forms, which are not only convenient to certain aspects of design, but also useful in understanding the basic physical principles of energy transfer. Consider the fluid velocities at the inlet and outlet of the turbomachine, again designated by the subscripts 1 and 2, respectively. By simple geometry,

$$C_{r2}^2 = C_2^2 - C_{w2}^2$$

and

$$C_{r2}^2 = V_2^2 - (U_2 - C_{w2})^2$$

Equating the values of C_{r2}^2 and expanding,

$$C_2^2 - C_{w2}^2 = V_2^2 - U_2^2 + 2U_2C_{w2} - C_{w2}^2$$

and

$$U_2C_{w2} = \frac{1}{2}\left(C_2^2 + U_2^2 - V_2^2\right)$$

Similarly,

$$U_1C_{w1} = \frac{1}{2}(C_1^2 + U_1^2 - V_1^2)$$

Inserting these values in the Euler equation,

$$E = \frac{1}{2}\left[(C_1^2 - C_2^2) + (U_1^2 - U_2^2) + (V_1^2 - V_2^2)\right] \tag{1.79}$$

The first term, $\frac{1}{2}(C_1^2 - C_2^2)$, represents the energy transfer due to change of absolute kinetic energy of the fluid during its passage between the entrance and exit sections. In a pump or compressor, the discharge kinetic energy from the rotor, $\frac{1}{2}C_2^2$, may be considerable. Normally, it is static head or pressure that is required as useful energy. Usually the kinetic energy at the rotor outlet is converted into a static pressure head by passing the fluid through a diffuser. In a turbine, the change in absolute kinetic energy represents the power transmitted from the fluid to the rotor due to an impulse effect. As this absolute kinetic energy change can be used to accomplish rise in pressure, it can be called a "virtual pressure rise" or "a pressure rise" which is possible to attain. The amount of pressure rise in the diffuser depends, of course, on the efficiency of the diffuser. Since this pressure rise comes from the diffuser, which is external to the rotor, this term, i.e., $\frac{1}{2}(C_1^2 - C_2^2)$, is sometimes called an "external effect."

The other two terms of Eq. (1.79) are factors that produce pressure rise within the rotor itself, and hence they are called "internal diffusion." The centrifugal effect, $\frac{1}{2}(U_1^2 - U_2^2)$, is due to the centrifugal forces that are developed as the fluid particles move outwards towards the rim of the machine. This effect is produced if the fluid changes radius as it flows from the entrance to the exit section. The third term, $\frac{1}{2}(V_1^2 - V_2^2)$, represents the energy transfer due to the change of the relative kinetic energy of the fluid. If $V_2 > V_1$, the passage acts like a nozzle and if $V_2 < V_1$, it acts like a diffuser. From the above discussions, it is apparent that in a turbocompresser, pressure rise occurs due to both external effects and internal diffusion effect. However, in axial flow compressors, the centrifugal effects are not utilized at all. This is why the pressure rise per stage is less than in a machine that utilizes all the kinetic energy effects available. It should be noted that the turbine derives power from the same effects.

Illustrative Example 1.1: A radial flow hydraulic turbine produces 32 kW under a head of 16 m and running at 100 rpm. A geometrically similar model producing 42 kW and a head of 6 m is to be tested under geometrically similar conditions. If model efficiency is assumed to be 92%, find the diameter ratio between the model and prototype, the volume flow rate through the model, and speed of the model.

Solution:

Assuming constant fluid density, equating head, flow, and power coefficients, using subscripts 1 for the prototype and 2 for the model, we have from Eq. (1.19),

$$\frac{P_1}{(\rho_1 N_1^3 D_1^5)} = \frac{P_2}{(\rho_2 N_2^3 D_2^5)}, \text{ where } \rho_1 = \rho_2.$$

Then, $\dfrac{D_2}{D_1} = \left(\dfrac{P_2}{P_1}\right)^{\frac{1}{5}} \left(\dfrac{N_1}{N_2}\right)^{\frac{3}{5}}$ or $\dfrac{D_2}{D_1} = \left(\dfrac{0.032}{42}\right)^{\frac{1}{5}} \left(\dfrac{N_1}{N_2}\right)^{\frac{3}{5}} = 0.238\left(\dfrac{N_1}{N_2}\right)^{\frac{3}{5}}$

Also, we know from Eq. (1.19) that

$$\frac{gH_1}{(N_1 D_1)^2} = \frac{gH_2}{(N_2 D_2)^2} \text{ (gravity remains constant)}$$

Then

$$\frac{D_2}{D_1} = \left(\frac{H_2}{H_1}\right)^{\frac{1}{2}} \left(\frac{N_1}{N_2}\right) = \left(\frac{6}{16}\right)^{\frac{1}{2}} \left(\frac{N_1}{N_2}\right)$$

Equating the diameter ratios, we get

$$0.238\left(\frac{N_1}{N_2}\right)^{\frac{3}{5}} = \left(\frac{6}{16}\right)^{\frac{1}{2}} \left(\frac{N_1}{N_2}\right)$$

or

$$\left(\frac{N_2}{N_1}\right)^{\frac{2}{5}} = \frac{0.612}{0.238} = 2.57$$

Therefore the model speed is

$$N_2 = 100 \times (2.57)^{\frac{5}{2}} = 1059 \text{ rpm}$$

Model scale ratio is given by

$$\frac{D_2}{D_1} = (0.238)\left(\frac{100}{1059}\right)^{\frac{3}{5}} = 0.238(0.094)^{0.6} = 0.058.$$

$$\text{Model efficiency is } \eta_m = \frac{\text{Power output}}{\text{Water power input}}$$

or,

$$0.92 = \frac{42 \times 10^3}{\rho g Q H},$$

or,

$$Q = \frac{42 \times 10^3}{0.92 \times 10^3 \times 9.81 \times 6} = 0.776 \ \mathrm{m^3/s}$$

Illustrative Example 1.2: A centrifugal pump delivers 2.5 m^3/s under a head of 14 m and running at a speed of 2010 rpm. The impeller diameter of the pump is 125 mm. If a 104 mm diameter impeller is fitted and the pump runs at a speed of 2210 rpm, what is the volume rate? Determine also the new pump head.

Solution:

First of all, let us assume that dynamic similarity exists between the two pumps. Equating the flow coefficients, we get [Eq. (1.3)]

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3} \quad \text{or} \quad \frac{2.5}{2010 \times (0.125)^3} = \frac{Q_2}{2210 \times (0.104)^3}$$

Solving the above equation, the volume flow rate of the second pump is

$$Q_2 = \frac{2.5 \times 2210 \times (0.104)^3}{2010 \times (0.125)^3} = 1.58 \ \mathrm{m^3/s}$$

Now, equating head coefficients for both cases gives [Eq. (1.9)]

$$gH_1/N_1^2 D_1^2 = gH_2/N_2^2 D_2^2$$

Substituting the given values,

$$\frac{9.81 \times 14}{(2010 \times 125)^2} = \frac{9.81 \times H_2}{(2210 \times 104)^2}$$

Therefore, $H_2 = 11.72$ m of water.

Illustrative Example 1.3: An axial flow compressor handling air and designed to run at 5000 rpm at ambient temperature and pressure of 18°C and 1.013 bar, respectively. The performance characteristic of the compressor is obtained at the atmosphere temperature of 25°C. What is the correct speed at which the compressor must run? If an entry pressure of 65 kPa is obtained at the point where the mass flow rate would be 64 kg/s, calculate the expected mass flow rate obtained in the test.

Solution:
Since the machine is the same in both cases, the gas constant R and diameter can be cancelled from the operating equations. Using first the speed parameter,

$$\frac{N_1}{\sqrt{T_{01}}} = \frac{N_2}{\sqrt{T_{02}}}$$

Therefore,

$$N_2 = 5000\left(\frac{273+25}{273+18}\right)^{\frac{1}{2}} = 5000\left(\frac{298}{291}\right)^{0.5} = 5060 \text{ rpm}$$

Hence, the correct speed is 5060 rpm. Now, considering the mass flow parameter,

$$\frac{m_1\sqrt{T_{01}}}{p_{01}} = \frac{m_2\sqrt{T_{02}}}{p_{02}}$$

Therefore,

$$m_2 = 64 \times \left(\frac{65}{101.3}\right)\left(\frac{291}{298}\right)^{0.5} = 40.58 \text{ kg/s}$$

Illustrative Example 1.4: A pump discharges liquid at the rate of Q against a head of H. If specific weight of the liquid is w, find the expression for the pumping power.

Solution:
Let Power P be given by:

$$P = f(w, Q, H) = kw^aQ^bH^c$$

where k, a, b, and c are constants. Substituting the respective dimensions in the above equation,

$$\mathrm{ML^2T^{-3}} = k(\mathrm{ML^{-2}T^{-2}})^a(\mathrm{L^3T^{-1}})^b(\mathrm{L})^c$$

Equating corresponding indices, for M, $1 = a$ or $a = 1$; for L, $2 = -2a + 3b + c$; and for T, $-3 = -2a - b$ or $b = 1$, so $c = 1$.

Therefore,

$$P = kwQH$$

Illustrative Example 1.5: Prove that the drag force F on a partially submerged body is given by:

$$F = V^2 l^2 \rho f\left(\frac{k}{l}, \frac{lg}{V^2}\right)$$

where V is the velocity of the body, l is the linear dimension, ρ, the fluid density, k is the rms height of surface roughness, and g is the gravitational acceleration.

Solution:

Let the functional relation be:

$$F = f(V, l, k, \rho, g)$$

Or in the general form:

$$F = f(F, V, l, k, \rho, g) = 0$$

In the above equation, there are only two primary dimensions. Thus, $m = 2$. Taking V, l, and ρ as repeating variables, we get:

$$\Pi_1 = (V)^{\mathrm{a}}(l)^{\mathrm{b}}(\rho)^{\mathrm{c}} F$$

$$\mathrm{M^o L^o T^o} = (\mathrm{LT}^{-1})^a (\mathrm{L})^b (\mathrm{ML}^{-3})^c (\mathrm{MLT}^{-2})$$

Equating the powers of M, L, and T on both sides of the equation, for M, $0 = c + 1$ or $c = -1$; for T, $0 = -a - 2$ or $a = -2$; and for L, $0 = a + b - 3c + 1$ or $b = -2$.

Therefore,

$$\Pi_1 = (V)^{-2}(l)^{-2}(\rho)^{-1} F = \frac{F}{V^2 l^2 \rho}$$

Similarly,

$$\Pi_2 = (V)^d (l)^e (\rho)^f (k)$$

Therefore,

$$\mathrm{M^0 L^0 T^0} = (\mathrm{LT}^{-1})^d (\mathrm{L})^e (\mathrm{ML}^{-3})^f (\mathrm{L})$$

for M, $0 = f$ or $f = 0$; for T, $0 = -d$ or $d = 0$; and for L, $0 = d + e - 3f + 1$ or $e = -1$.

Thus,

$$\Pi_2 = (V)^0 (l)^{-1} (\rho)^0 k = \frac{k}{l}$$

and

$$\Pi_3 = (V)^g (l)^h (\rho)^i (g)$$

$$M^0L^0T^0 = (LT^{-1})^g (L)^h (ML^{-3})^i (LT^{-2})$$

Equating the exponents gives, for M, $0 = i$ or $i = 0$; for T, $0 = -g-2$ or $g = -2$; for L, $0 = g + h - 3i + 1$ or $h = 1$.

$$\text{Therefore,}\quad \Pi_3 = V^{-2} l^1 \rho^0 g = \frac{lg}{V^2}$$

Now the functional relationship may be written as:

$$f\left(\frac{F}{V^2 l^2 \rho}, \frac{k}{l}, \frac{lg}{V^2}\right) = 0$$

Therefore,

$$F = V^2 l^2 \rho f\left(\frac{k}{l}, \frac{lg}{V^2}\right)$$

Illustrative Example 1.6: Consider an axial flow pump, which has rotor diameter of 32 cm that discharges liquid water at the rate of 2.5 m^3/min while running at 1450 rpm. The corresponding energy input is 120 J/kg, and the total efficiency is 78%. If a second geometrically similar pump with diameter of 22 cm operates at 2900 rpm, what are its (1) flow rate, (2) change in total pressure, and (3) input power?

Solution:

Using the geometric and dynamic similarity equations,

$$\frac{Q_1}{N_1 D_1^2} = \frac{Q_2}{N_2 D_2^2}$$

Therefore,

$$Q_2 = \frac{Q_1 N_2 D_2^2}{N_1 D_1^2} = \frac{(2.5)(2900)(0.22)^2}{(1450)(0.32)^2} = 2.363 \text{ m}^3/\text{min}$$

As the head coefficient is constant,

$$W_2 = \frac{W_1 N_2^2 D_2^2}{N_1^2 D_1^2} = \frac{(120)(2900)^2(0.22)^2}{(1450)^2(0.32)^2} = 226.88 \text{ J/kg}$$

The change in total pressure is:

$$\Delta P = W_2 \eta_{tt} \rho = (226.88)(0.78)(1000) \text{ N/m}^2$$

$$= (226.88)(0.78)(1000)10^{-5} = 1.77 \text{ bar}$$

Input power is given by

$$P = \dot{m}W_2 = \frac{(1000)(2.363)(0.22688)}{60} = 8.94\ \text{kW}$$

Illustrative Example 1.7: Consider an axial flow gas turbine in which air enters at the stagnation temperature of 1050 K. The turbine operates with a total pressure ratio of 4:1. The rotor turns at 15500 rpm and the overall diameter of the rotor is 30 cm. If the total-to-total efficiency is 0.85, find the power output per kg per second of airflow if the rotor diameter is reduced to 20 cm and the rotational speed is 12,500 rpm. Take $\gamma = 1.4$.

Solution:
Using the isentropic P–T relation:

$$T_{02}^{'} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{(\gamma-1)}{2}} = (1050)\left(\frac{1}{4}\right)^{0.286} = 706.32\text{K}$$

Using total-to-total efficiency,

$$T_{01} - T_{02} = \left(T_{01} - T_{02}'\right)\eta_{\text{tt}} = (343.68)(0.85) = 292.13\ \text{K}$$

and

$$W_1 = c_{\text{p}}\Delta T_0 = (1.005)(292.13) = 293.59\ \text{kJ/kg}$$

$$W_2 = \frac{W_1 N_2^2 D_2^2}{N_1^2 D_1^2} = \frac{(293.59 \times 10^3)(12,500)^2(0.20)^2}{(15,500)^2(0.30)^2}$$

$$= 84,862\ \text{J/kg}$$

$\therefore$ Power output $= 84.86$ kJ/kg

Illustrative Example 1.8: At what velocity should tests be run in a wind tunnel on a model of an airplane wing of 160 mm chord in order that the Reynolds number should be the same as that of the prototype of 1000 mm chord moving at 40.5 m/s. Air is under atmospheric pressure in the wind tunnel.

Solution:
Let

Velocity of the model: V_{m}

Length of the model: $L_{\text{m}} = 160$ mm

Length of the prototype: $L_{\text{p}} = 1000$ mm

Velocity of the prototype: $V_{\text{p}} = 40.5$ m/s

According to the given conditions:

$$(\mathrm{Re})_\mathrm{m} = (\mathrm{Re})_\mathrm{p}$$

$$\frac{V_\mathrm{m} L_\mathrm{m}}{\nu_\mathrm{m}} = \frac{V_\mathrm{p} L_\mathrm{p}}{\nu_\mathrm{p}}, \text{ Therefore, } \nu_\mathrm{m} = \nu_\mathrm{p} = \nu_\mathrm{air}$$

Hence

$$V_\mathrm{m} L_\mathrm{m} = V_\mathrm{p} L_\mathrm{p},$$

or

$$V_\mathrm{m} = L_\mathrm{p} V_\mathrm{p} / L_\mathrm{m} = 40.5 \times 1000/160 = 253.13 \text{ m/s}$$

Illustrative Example 1.9: Show that the kinetic energy of a body equals kmV^2 using the method of dimensional analysis.

Solution:
Since the kinetic energy of a body depends on its mass and velocity,

$$\text{K.E.} = f(V, m), \text{ or K.E.} = \mathrm{k} V^a m^b.$$

Dimensionally,

$$\mathrm{FLT}^0 = (\mathrm{LT}^{-1})^a (\mathrm{FT}^2 \mathrm{L}^{-1})^b$$

Equating the exponents of F, L, and T, we get:

$$\text{F: } 1 = b; \quad \text{L: } 1 = a - b; \quad \text{T: } 0 = -a + 2b$$

This gives $b = 1$ and $a = 2$. So, K.E. $= kV^2 m$, where k is a constant.

Illustrative Example 1.10: Consider a radial inward flow machine, the radial and tangential velocity components are 340 m/s and 50 m/s, respectively, and the inlet and the outlet radii are 14 cm and 7 cm, respectively. Find the torque per unit mass flow rate.

Solution:
Here,

$$r_1 = 0.14 \text{ m}$$

$$C_{w1} = 340 \text{ m/s},$$

$$r_2 = 0.07 \text{ m}$$

$$C_{w2} = 50 \text{ m/s}$$

Torque is given by:

$$T = r_1 C_{w1} - r_2 C_{w2}$$
$$= (0.14 \times 340 - 0.07 \times 50)$$
$$= (47.6 - 3.5) = 44.1 \text{ N-m per kg/s}$$

PROBLEMS

1.1 Show that the power developed by a pump is given by

$$P = kwQH$$

where $k =$ constant, $w =$ specific weight of liquid, $Q =$ rate of discharge, and $H =$ head dimension.

1.2 Develop an expression for the drag force on a smooth sphere of diameter D immersed in a liquid (of density ρ and dynamic viscosity μ) moving with velocity V.

1.3 The resisting force F of a supersonic plane in flight is given by:

$$F = f(L, V, \rho, \mu, k)$$

where $L =$ the length of the aircraft, $V =$ velocity, $\rho =$ air density, $\mu =$ air viscosity, and $k =$ the bulk modulus of air.

1.4 Show that the resisting force is a function of Reynolds number and Mach number.

1.5 The torque of a turbine is a function of the rate of flow Q, head H, angular velocity ω, specific weight w of water, and efficiency. Determine the torque equation.

1.6 The efficiency of a fan depends on density ρ, dynamic viscosity μ of the fluid, angular velocity ω, diameter D of the rotor and discharge Q. Express efficiency in terms of dimensionless parameters.

1.7 The specific speed of a Kaplan turbine is 450 when working under a head of 12 m at 150 rpm. If under this head, 30,000 kW of energy is generated, estimate how many turbines should be used.

(7 turbines).

1.8 By using Buckingham's Π theorem, show that dimensionless expression ΔP is given by:

$$\Delta P = \frac{4 f V^2 \rho l}{2D}$$

where $\triangle P =$ pressure drop in a pipe, $V =$ mean velocity of the flow, $l =$ length of the pipe, $D =$ diameter of the pipe, $\mu =$ viscosity of the fluid, $k =$ average roughness of the pipe, and $\rho =$ density of the fluid.

1.9 If H_{f} is the head loss due to friction ($\triangle P/w$) and w is the specific weight of the fluid, show that

$$H_f = \frac{4f V^2 l}{2gD}$$

(other symbols have their usual meaning).

1.10 Determine the dimensions of the following in M.L.T and F.L.T systems: (1) mass, (2) dynamic viscosity, and (3) shear stress.

$$\left(\mathrm{M}, \mathrm{FT^2L^{-1}}, \mathrm{ML^{-1}T^{-1}}, \mathrm{FTL^{-2}}, \mathrm{ML^{-1}T^{-2}}, \mathrm{FL^{-3}}\right)$$

NOTATION

A_{r}	area ratio
a	sonic velocity
B_{r}	breadth of prototype
C	velocity of gas, absolute velocity of turbo machinery
D	diameter of pipe, turbine runner, or pump
D_{p}	depth of the prototype
E	energy transfer by a rotor or absorbed by the rotor
F	force
F_{r}	force ratio
g	local acceleration due to gravity
H	head
h	specific enthalpy
h_0	stagnation enthalpy
K.E.	kinetic energy
L	length
L_{p}	length of prototype
L_{r}	scale ratio
M	Mach number
m	mass rate of flow
N	speed
N_{s}	specific speed
P	power
P_{h}	hydraulic power
P_{m}	power loss due to mechanical friction at the bearing
P_{s}	shaft power
P.E.	potential energy

p	fluid pressure
p_0	stagnation pressure
Q	volume rate of flow, heat transfer
R	gas constant
Re	Reynolds number
r	radius of rotor
s	specific entropy
sp . gr	specific gravity of fluid
T	temperature, time
T_0	stagnation temperature
t	time
U	rotor speed
V	relative velocity, mean velocity
W	work
V_r	volume ratio, velocity ratio
W_t	actual turbine work output
W_t'	isentropic turbine work output
α	absolute air angle
β	relative air angle
γ	specific weight, specific heat ratio
η	efficiency
$\eta_{\propto \mathrm{c}}$	polytropic efficiency of compressor
$\eta_{\propto \mathrm{t}}$	polytropic efficiency of turbine
η_c	compressor efficiency
η_d	diffuser efficiency
η_h	hydraulic efficiency
η_j	jet pipe or nozzle efficiency
η_m	mechanical efficiency
η_o	overall efficiency
η_p	prototype efficiency
η_s	isentropic efficiency
η_t	turbine efficiency
η_ts	total-to-static efficiency
η_tt	total-to-total efficiency
η_v	volumetric efficiency
μ	absolute or dynamic viscosity
ν	kinematic viscosity
Π	dimensionless parameter
ρ	mass density
τ	shear stress, torque exerted by or acting on the rotor
ω	angular velocity

SUFFIXES

0	stagnation conditions
1	inlet to rotor
2	outlet from the rotor
3	outlet from the diffuser
a	axial
h	hub
r	radial
t	tip
w	whirl or tangential