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## Hydraulic Pumps

### 2.1 INTRODUCTION

Hydraulics is defined as the science of the conveyance of liquids through pipes. The pump is often used to raise water from a low level to a high level where it can be stored in a tank. Most of the theory applicable to hydraulic pumps has been derived using water as the working fluid, but other liquids can also be used. In this chapter, we will assume that liquids are totally incompressible unless otherwise specified. This means that the density of liquids will be considered constant no matter how much pressure is applied. Unless the change in pressure in a particular situation is very great, this assumption will not cause a significant error in calculations. Centrifugal and axial flow pumps are very common hydraulic pumps. Both work on the principle that the energy of the liquid is increased by imparting kinetic energy to it as it flows through the pump. This energy is supplied by the impeller, which is driven by an electric motor or some other drive. The centrifugal and axial flow pumps will be discussed separately in the following sections.

### 2.2 CENTRIFUGAL PUMPS

The three important parts of centrifugal pumps are (1) the impeller, (2) the volute casing, and (3) the diffuser.

### 2.2.1 Impeller

The centrifugal pump is used to raise liquids from a lower to a higher level by creating the required pressure with the help of centrifugal action. Whirling motion is imparted to the liquid by means of backward curved blades mounted on a wheel known as the impeller. As the impeller rotates, the fluid that is drawn into the blade passages at the impeller inlet or eye is accelerated as it is forced radially outwards. In this way, the static pressure at the outer radius is much higher than at the eye inlet radius. The water coming out of the impeller is then lead through the pump casing under high pressure. The fluid has a very high velocity at the outer radius of the impeller, and, to recover this kinetic energy by changing it into pressure energy, diffuser blades mounted on a diffuser ring may be used. The stationary blade passages have an increasing cross-sectional area. As the fluid moves through them, diffusion action takes place and hence the kinetic energy is converted into pressure energy. Vaneless diffuser passages may also be used. The fluid moves from the diffuser blades into the volute casing. The functions of a volute casing can be summarized as follows: It collects water and conveys it to the pump outlet. The shape of the casing is such that its area of cross-section gradually increases towards the outlet of the pump. As the flowing water progresses towards the delivery pipe, more and more water is added from the outlet periphery of the impeller. [Figure 2.1](#) shows a centrifugal pump impeller with the velocity triangles at inlet and outlet.

For the best efficiency of the pump, it is assumed that water enters the impeller radially, i.e.,  $\alpha_1 = 90^\circ$  and  $C_{w1} = 0$ . Using Euler's pump equation, the work done per second on the water per unit mass of fluid flowing

$$E = \frac{W}{m} = (U_2 C_{w2} - U_1 C_{w1}) \quad (2.1)$$

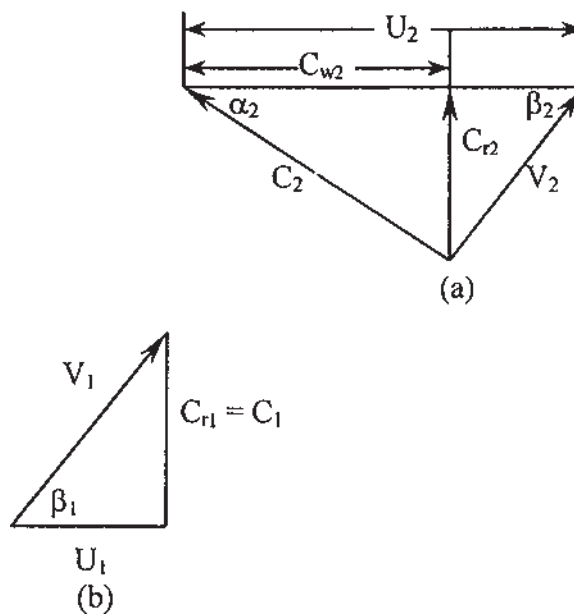
Where  $C_w$  is the component of absolute velocity in the tangential direction.  $E$  is referred to as the Euler head and represents the ideal or theoretical head developed by the impeller only. The flow rate is

$$Q = 2\pi r_1 C_{r1} b_1 = 2\pi r_2 C_{r2} b_2 \quad (2.2)$$

Where  $C_r$  is the radial component of absolute velocity and is perpendicular to the tangent at the inlet and outlet and  $b$  is the width of the blade. For shockless entry and exit to the vanes, water enters and leaves the vane tips in a direction parallel to their relative velocities at the two tips.

As discussed in [Chapter 1](#), the work done on the water by the pump consists of the following three parts:

1. The part  $(C_2^2 - C_1^2)/2$  represents the change in kinetic energy of the liquid.
2. The part  $(U_2^2 - U_1^2)/2$  represents the effect of the centrifugal head or energy produced by the impeller.



**Figure 2.1** Velocity triangles for centrifugal pump impeller.

3. The part  $(V_2^2 - V_1^2)/2$  represents the change in static pressure of the liquid, if the losses in the impeller are neglected.

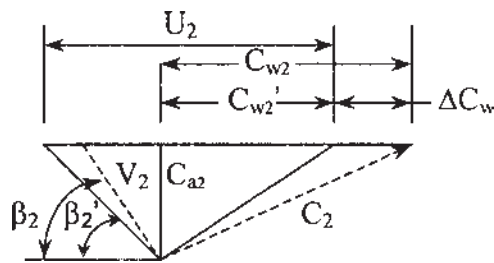
### 2.3 SLIP FACTOR

From the preceding section, it may be seen that there is no assurance that the actual fluid will follow the blade shape and leave the impeller in a radial direction. There is usually a slight slippage of the fluid with respect to the blade rotation. **Figure 2.2** shows the velocity triangles at impeller tip.

In **Fig. 2.2**,  $\beta_2'$  is the angle at which the fluid leaves the impeller, and  $\beta_2$  is the actual blade angle, and  $C_{w2}$  and  $C_{w2}'$  are the tangential components of absolute velocity corresponding to the angles  $\beta_2$  and  $\beta_2'$ , respectively. Thus,  $C_{w2}$  is reduced to  $C_{w2}'$  and the difference  $\Delta C_w$  is defined as the slip. The slip factor is defined as

$$\text{Slip factor, } \sigma = \frac{C_{w2}'}{C_{w2}}$$

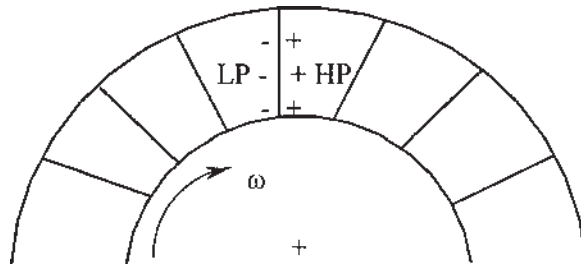
According to Stodola's theory, slip in centrifugal pumps and compressors is due to relative rotation of fluid in a direction opposite to that of impeller with the same



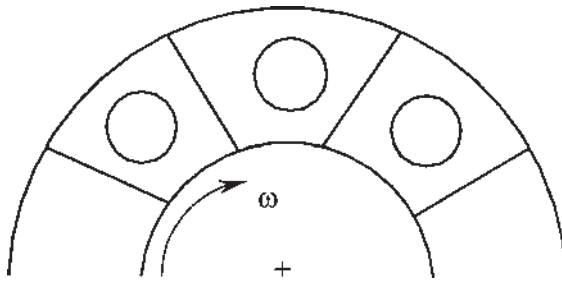
**Figure 2.2** Velocity triangle at impeller outlet with slip.

angular velocity as that of an impeller. Figure 2.3 shows the leading side of a blade, where there is a high-pressure region while on the trailing side of the blade there is a low-pressure region.

Due to the lower pressure on the trailing face, there will be a higher velocity and a velocity gradient across the passage. This pressure distribution is associated with the existence of circulation around the blade, so that low velocity on the high-pressure side and high velocity on the low-pressure side and velocity distribution is not uniform at any radius. Due to this fact, the flow may separate from the suction surface of the blade. Thus,  $C_{w2}$  is less than  $C_{w2}'$  and the difference is defined as the slip. Another way of looking at this effect, as given by Stodola, is shown in Fig. 2.4, the impeller itself has an angular velocity  $\omega$  so that, relative to the impeller, the fluid must have an angular velocity of  $-\omega$ ; the result of this being a circulatory motion relative to the channel or relative eddy. The net result of the previous discussion is that the fluid is discharged from the impeller at an angle relative to the impeller, which is less than the vane angle as mentioned earlier.



**Figure 2.3** Pressure distribution on impeller vane. LP = low pressure, HP = high pressure.



**Figure 2.4** Relative eddy in impeller channel.

Hence, the slip factor  $\sigma$  is defined as

$$\sigma = \frac{C'_{w2}}{C_{w2}} \quad (2.3)$$

For purely radial blades, which are often used in centrifugal compressors,  $\beta_2$  will be  $90^\circ$  and the Stodola slip factor becomes

$$\sigma = 1 - \frac{\pi}{n} \quad (2.4)$$

where  $n$  is the number of vanes. The Stanitz slip factor is given by

$$\sigma = 1 - \frac{0.63\pi}{n} \quad (2.5)$$

When applying a slip factor, the Euler pump equation becomes

$$\frac{W}{m} = \sigma U_2 C_{w2} - U_1 C_{w1} \quad (2.6)$$

Typically, the slip factor lies in the region of 0.9, while the slip occurs even if the fluid is ideal.

## 2.4 PUMP LOSSES

The following are the various losses occurring during the operation of a centrifugal pump.

1. Eddy losses at entrance and exit of impeller, friction losses in the impeller, frictional and eddy losses in the diffuser, if provided.
2. Losses in the suction and delivery pipe. The above losses are known as hydraulic losses.
3. Mechanical losses are losses due to friction of the main bearings, and stuffing boxes. Thus, the energy supplied by the prime mover to

impeller is equal to the energy produced by impeller plus mechanical losses. A number of efficiencies are associated with these losses.

Let  $\rho$  be the density of liquid;  $Q$ , flow rate;  $H$ , total head developed by the pump;  $P_s$ , shaft power input;  $H_i$ , total head across the impeller; and  $h_i$ , head loss in the impeller. Then, the overall efficiency  $\eta_o$  is given by:

$$\eta_o = \frac{\text{Fluid power developed by pump}}{\text{Shaft power input}} = \frac{\rho g Q H}{P_s} \quad (2.7)$$

Casing efficiency  $\eta_c$  is given by:

$$\begin{aligned} \eta_c &= \text{Fluid power at casing outlet/fluid power at casing inlet} \\ &= \text{Fluid power at casing outlet}/(\text{fluid power developed by} \\ &\quad \text{impeller} - \text{leakage loss}) \\ &= \rho g Q H / \rho g Q H_i = H / H_i \end{aligned} \quad (2.8)$$

Impeller efficiency  $\eta_i$  is given by:

$$\begin{aligned} \eta_i &= \text{Fluid power at impeller exit/fluid} \\ &\quad \text{power supplied to impeller} \\ &= \text{Fluid power at impeller exit}/(\text{fluid power} \\ &\quad \text{developed by impeller} \\ &\quad + \text{impeller loss}) \\ &= \rho g Q_i H_i / [\rho g Q_i (H_i + h_i)] = H_i / (H_i + h_i) \end{aligned} \quad (2.9)$$

Volumetric efficiency  $\eta_v$  is given by:

$$\begin{aligned} \eta_v &= \text{Flow rate through pump}/\text{flow rate through impeller} \\ &= Q / (Q + q) \end{aligned} \quad (2.10)$$

Mechanical efficiency  $\eta_m$  is given by:

$$\begin{aligned} \eta_m &= \text{Fluid power supplied to the impeller}/\text{power} \\ &\quad \text{input to the shaft} \\ &= \rho g Q_i (h_i + H_i) / P_s \end{aligned} \quad (2.11)$$

Therefore,

$$\eta_o = \eta_c \eta_i \eta_v \eta_m \quad (2.12)$$

A hydraulic efficiency may be defined as

$$\begin{aligned}\eta_h &= \frac{\text{Actual head developed by pump}}{\text{Theoretical head developed by impeller}} \\ &= \frac{H}{(H_1 + h_i)}\end{aligned}\quad (2.13)$$

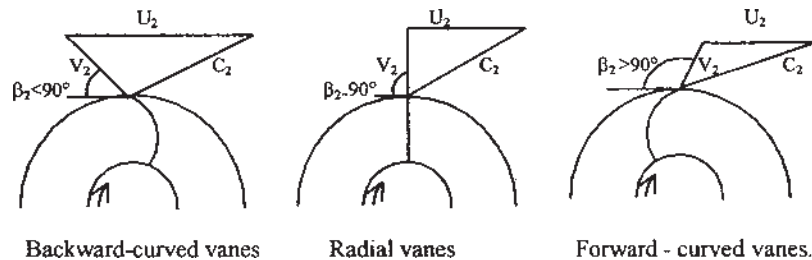
The head  $H$  is also known as manometric head.

## 2.5 THE EFFECT OF IMPELLER BLADE SHAPE ON PERFORMANCE

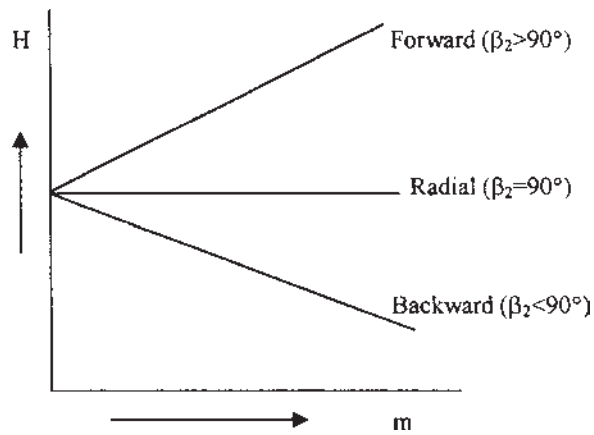
The various blade shapes utilized in impellers of centrifugal pumps/compressors are shown in Fig. 2.5. The blade shapes can be classified as:

1. Backward-curved blades ( $\beta_2 < 90^\circ$ )
2. Radial blades ( $\beta_2 = 90^\circ$ )
3. Forward-curved blades ( $\beta_2 > 90^\circ$ )

As shown in Fig. 2.5, for backward-curved vanes, the value of  $C_{w2}$  (whirl component at outlet) is much reduced, and thus, such rotors have a low energy transfer for a given impeller tip speed, while forward-curved vanes have a high value of energy transfer. Therefore, it is desirable to design for high values of  $\beta_2$  (over  $90^\circ$ ), but the velocity diagrams show that this also leads to a very high value of  $C_2$ . High kinetic energy is seldom required, and its reduction to static pressure by diffusion in a fixed casing is difficult to perform in a reasonable sized casing. However, radial vanes ( $\beta_2 = 90^\circ$ ) have some particular advantages for very high-speed compressors where the highest possible pressure is required. Radial vanes are relatively easy to manufacture and introduce no complex bending stresses (Fig. 2.6).



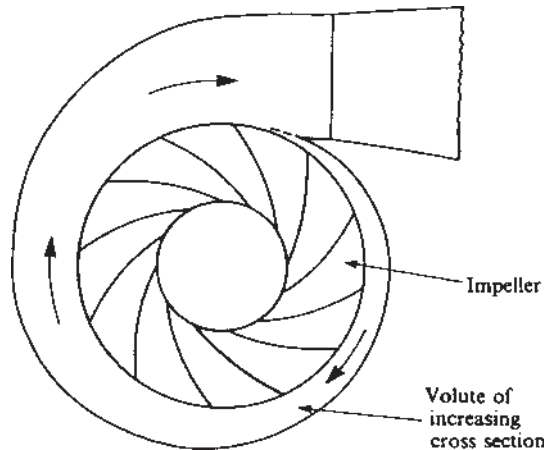
**Figure 2.5** Centrifugal pump outlet velocity triangles for varying blade outlet angle.



**Figure 2.6** Characteristics for varying outlet blade angle.

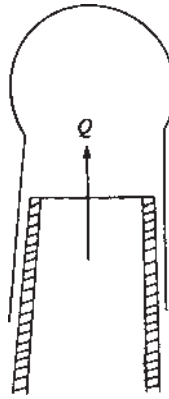
## 2.6 VOLUTE OR SCROLL COLLECTOR

A volute or scroll collector consists of a circular passage of increasing cross-sectional area (Fig. 2.7). The advantage of volute is its simplicity and low cost. The cross-sectional area increases as the increment of discharge increases around the periphery of the impeller, and, if the velocity is constant in the volute,



**Figure 2.7** Volute or scroll collector.





**Figure 2.8** Cross-section of volute casing.

then the static pressure is likewise constant and the radial thrust will be zero. Any deviation in capacity (i.e., flow rate) from the design condition will result in a radial thrust which if allowed to persist could result in shaft bending.

The cross-sectional shape of the volute is generally similar to that shown in Fig. 2.8, with the sidewalls diverging from the impeller tip and joined by a semicircular outer wall. The circular section is used to reduce the losses due to friction and impact when the fluid hits the casing walls on exiting from the impeller.

## 2.7 VANELESS DIFFUSER

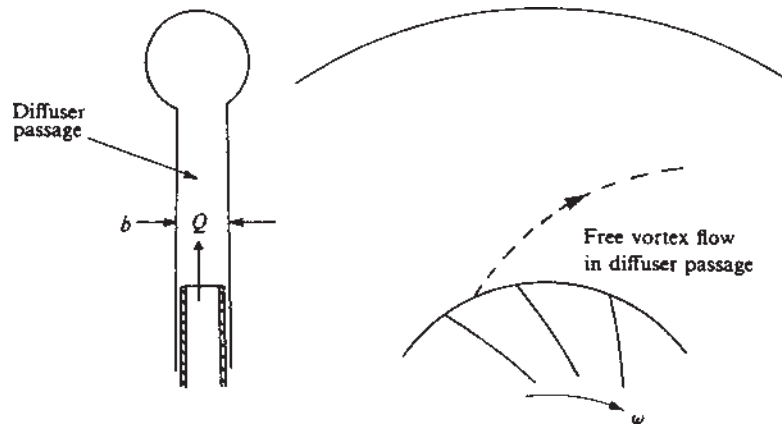
For the diffusion process, the vaneless diffuser is reasonably efficient and is best suited for a wide range of operations. It consists simply of an annular passage without vanes surrounding the impeller. A vaneless diffuser passage is shown in Fig. 2.9. The size of the diffuser can be determined by using the continuity equation. The mass flow rate in any radius is given by

$$m = \rho A C_r = 2\pi r b \rho C_r \quad (2.14)$$

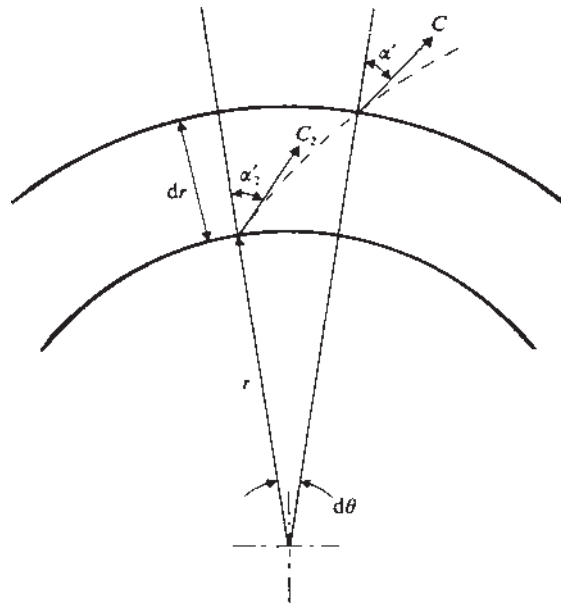
where  $b$  is the width of the diffuser passage,

$$C_r = \frac{r_2 b_2 \rho_2 C_{r2}}{r b \rho} \quad (2.15)$$

where subscripted variables represent conditions at the impeller outlet and the unsubscripted variables represent conditions at any radius  $r$  in the vaneless diffuser. Assuming the flow is frictionless in the diffuser, angular momentum



**Figure 2.9** Vaneless diffuser.



**Figure 2.10** Logarithmic spiral flow in vaneless space.

is constant and

$$C_w = (C_{w2}r_2)/r \quad (2.16)$$

But the tangential velocity component ( $C_w$ ) is usually very much larger than the radial velocity component  $C_r$ , and, therefore, the ratio of the inlet to outlet diffuser velocities  $\frac{C_2}{C_3} = \frac{r_3}{r_2}$ .

It means that for a large reduction in the outlet kinetic energy, a diffuser with a large radius is required. For an incompressible flow,  $rC_r$  is constant, and, therefore,  $\tan \alpha = C_w/C_r = \text{constant}$ . Thus, the flow maintains a constant inclination to radial lines, the flow path traces a logarithmic spiral.

As shown in Fig. 2.10, for an incremental radius  $dr$ , the fluid moves through angle  $d\theta$ , then  $rd\theta = dr \tan \alpha$ .

Integrating we have

$$\theta - \theta_2 = \tan \alpha \log(r/r_2) \quad (2.17)$$

Substituting  $\alpha = 78^\circ$  and  $(r/r_2) = 2$ , the change in angle of the diffuser is equal to  $180^\circ$ . Because of the long flow path with this type of diffuser, friction effects are high and the efficiency is low.

## 2.8 VANED DIFFUSER

The vaned diffuser is advantageous where small size is important. In this type of diffuser, vanes are used to diffuse the outlet kinetic energy of the fluid at a much higher rate than is possible by a simple increase in radius, and hence it is possible to reduce the length of flow path and diameter. The vane number, the angle of divergence is smaller, and the diffuser becomes more efficient, but greater is the friction. The cross section of the diffuser passage should be square to give a maximum hydraulic radius. However, the number of diffuser vanes should have no common factor with the number of impeller vanes. The collector and diffuser operate at their maximum efficiency at the design point only. Any deviation from the design discharge will change the outlet velocity triangle and the subsequent flow in the casing.

## 2.9 CAVITATION IN PUMPS

Cavitation is caused by local vaporization of the fluid, when the local static pressure of a liquid falls below the vapor pressure of the liquid. Small bubbles or cavities filled with vapor are formed, which suddenly collapse on moving forward with the flow into regions of high pressure. These bubbles collapse with tremendous force, giving rise to pressure as high as 3500 atm. In a centrifugal pump, these low-pressure zones are generally at the impeller inlet, where the fluid is locally accelerated over the vane surfaces. In turbines, cavitation is most likely

to occur at the downstream outlet end of a blade on the low-pressure leading face. When cavitation occurs, it causes the following undesirable effects:

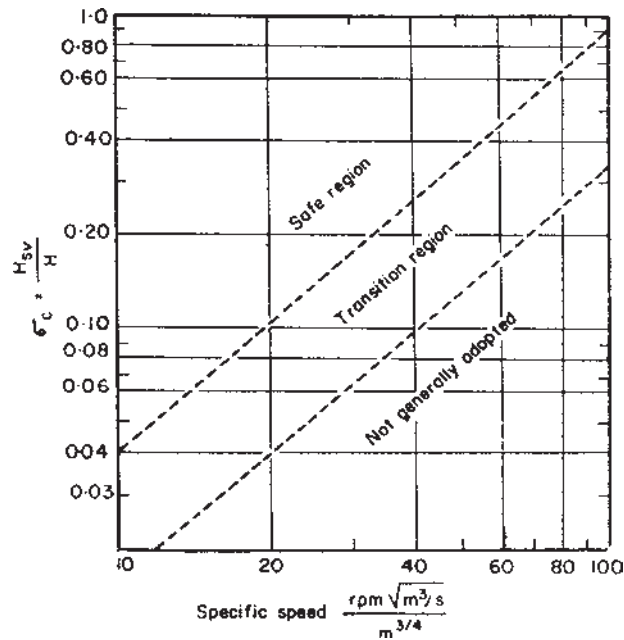
1. Local pitting of the impeller and erosion of the metal surface.
2. Serious damage can occur from this prolonged cavitation erosion.
3. Vibration of machine and noise is also generated in the form of sharp cracking sounds when cavitation takes place.
4. A drop in efficiency due to vapor formation, which reduces the effective flow areas.

The avoidance of cavitation in conventionally designed machines can be regarded as one of the essential tasks of both pump and turbine designers. This cavitation imposes limitations on the rate of discharge and speed of rotation of the pump.

A cavitation parameter is defined as  $\sigma_c = \text{pump total inlet head above vapor pressure/head developed by the pump or at the inlet flange}$

$$\sigma_c = \left( \frac{p_1}{\rho g} + \frac{V_1^2}{2g} - \frac{p_v}{\rho g} \right) / H \quad (2.18)$$

The numerator of Eq. (2.18) is a suction head and is called the net positive suction



**Figure 2.11** Cavitation limits for radial flow pumps.

head (NPSH) of the pump. It is a measure of the energy available on the suction side of the pump, and  $H$  is the manometric head. The cavitation parameter is a function of specific speed, efficiency of the pump, and number of vanes. [Figure 2.11](#) shows the relationship between  $\sigma_c$  and  $N_s$ . It may be necessary in the selection of pumps that the value of  $\sigma_c$  does not fall below the given value by the plots in [Fig. 2.11](#) for any condition of operation.

## 2.10 SUCTION SPECIFIC SPEED

The efficiency of the pump is a function of flow coefficient and suction specific speed, which is defined as

$$N_{\text{suc}} = NQ^{1/2} [g(\text{NPSH})]^{-3/4}$$

Thus,

$$\eta = f(Q, N_{\text{suc}})$$

The cavitation parameter may also be determined by the following equation

$$\begin{aligned} N_s/N_{\text{suc}} &= (\text{NPSH})^{3/4}/H^{3/4} \\ &= \sigma_c^{3/4} \end{aligned} \quad (2.19)$$

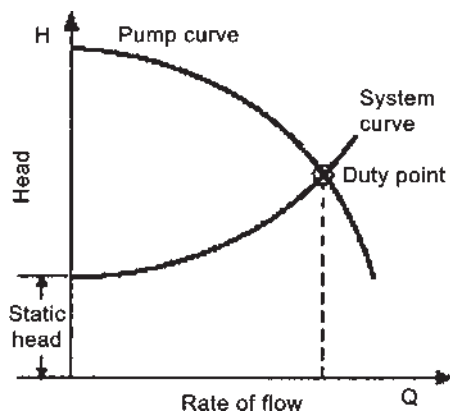
## 2.11 AXIAL FLOW PUMP

In an axial flow pump, pressure is developed by flow of liquid over blades of airfoil section. It consists of a propeller-type of impeller running in a casing. The advantage of an axial flow pump is its compact construction as well as its ability to run at extremely high speeds. The flow area is the same at inlet and outlet and the minimum head for this type of pump is the order of 20 m.

## 2.12 PUMPING SYSTEM DESIGN

Proper pumping system design is the most important single element in minimizing the life cycle cost. All pumping systems are comprised of a pump, a driver, pipe installation, and operating controls. Proper design considers the interaction between the pump and the rest of the system and the calculation of the operating duty point(s) ([Fig. 2.12](#)). The characteristics of the piping system must be calculated in order to determine required pump performance. This applies to both simple systems as well as to more complex (branched) systems.

Both procurement costs and the operational costs make up the total cost of an installation during its lifetime. A number of installation and operational costs are directly dependent on the piping diameter and the components in the piping system.



**Figure 2.12** The duty point of the pump is determined by the intersection of the system curve and the pump curve as shown above.

A considerable amount of the pressure losses in the system are caused by valves, in particular, control valves in throttle-regulated installations. In systems with several pumps, the pump workload is divided between the pumps, which together, and in conjunction with the piping system, deliver the required flow.

The piping diameter is selected based on the following factors:

- Economy of the whole installation (pumps and system)
- Required lowest flow velocity for the application (e.g., avoid sedimentation)
- Required minimum internal diameter for the application (e.g., solid handling)
- Maximum flow velocity to minimize erosion in piping and fittings
- Plant standard pipe diameters.

Decreasing the pipeline diameter has the following effects:

- Piping and component procurement and installation costs will decrease.
- Pump installation procurement costs will increase as a result of increased flow losses with consequent requirements for higher head pumps and larger motors. Costs for electrical supply systems will therefore increase.
- Operating costs will increase as a result of higher energy usage due to increased friction losses.

Some costs increase with increasing pipeline size and some decrease. Because of this, an optimum pipeline size may be found, based on minimizing costs over the life of the system. A pump application might need to cover several

duty points, of which the largest flow and/or head will determine the rated duty for the pump. The pump user must carefully consider the duration of operation at the individual duty points to properly select the number of pumps in the installation and to select output control.

### **2.12.1 Methods for Analyzing Existing Pumping Systems**

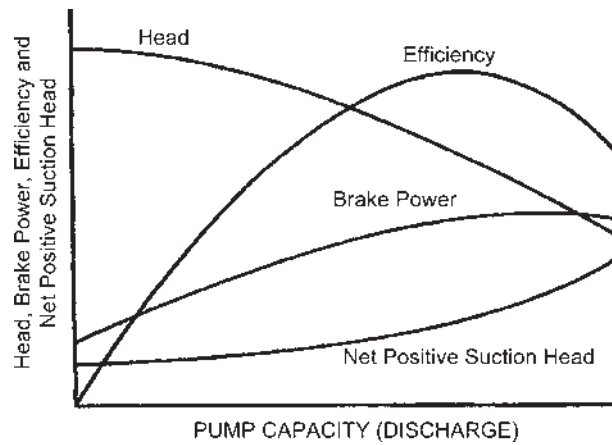
The following steps provide an overall guideline to improve an existing pumping system.

- Assemble a complete document inventory of the items in the pumping system.
- Determine the flow rates required for each load in the system.
- Balance the system to meet the required flow rates of each load.
- Minimize system losses needed to balance the flow rates.
- Affect changes to the pump to minimize excessive pump head in the balanced system.
- Identify pumps with high maintenance cost.

One of two methods can be used to analyze existing pumping systems. One consists of observing the operation of the actual piping system, and the second consists of performing detailed calculations using fluid analysis techniques. The first method relies on observation of the operating piping system (pressures, differential pressures, and flow rates), the second deals with creating an accurate mathematical model of the piping system and then calculating the pressure and flow rates with the model.

The following is a checklist of some useful means to reduce the life cycle cost of a pumping system.

- Consider all relevant costs to determine the life cycle cost.
- Procure pumps and systems using life cycle cost considerations.
- Optimize total cost by considering operational costs and procurement costs.
- Consider the duration of the individual pump duty points.
- Match the equipment to the system needs for maximum benefit.
- Match the pump type to the intended duty.
- Do not oversize the pump.
- Match the driver type to the intended duty.
- Specify motors to have high efficiency.
- Monitor and sustain the pump and the system to maximize benefit.
- Consider the energy wasted using control valves.



**Figure 2.13** Typical pump characteristics.

### 2.12.2 Pump System Interaction

*The actual operating point on the pump system characteristic curve is defined by its interaction with the operating characteristics of the installed system (Fig. 2.13).*

The system characteristics will consist of:

- The total static head, being the difference in elevation between the upstream and downstream controls (generally represented by reservoir levels),
- The energy losses in the system (generally pipe friction), which are normally expressed as a function of velocity head.
- The interaction point of these curves represents the actual operating point (as shown later), defining the *Head* supplied by the pump and the *Discharge* of the system. The efficiency of the pump under these conditions will also be defined.

Note that the efficiency of the pump at this operating point is the critical parameter in pump selection and suitability for a particular system (Figs. 2.14 and 2.15).

## 2.13 LIFE CYCLE ANALYSIS

Over the economic life of a pumped supply system, a number of design parameter will change. System behavior for all possible operating environments is needed (Fig. 2.16). Parameters that will change include:



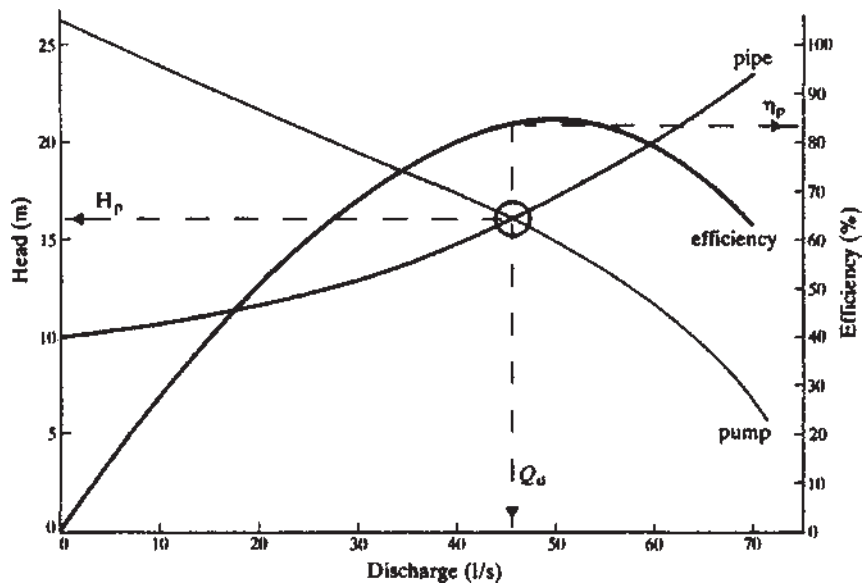


Figure 2.14 Pump–system interaction point and pump efficiency.

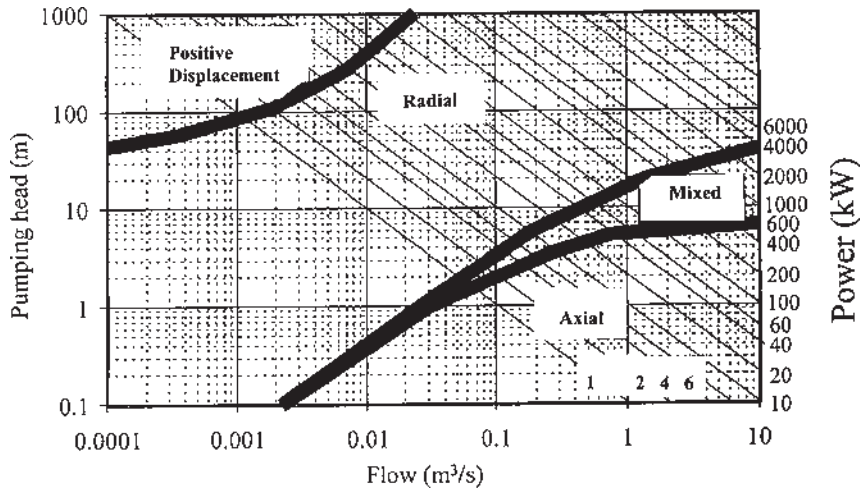
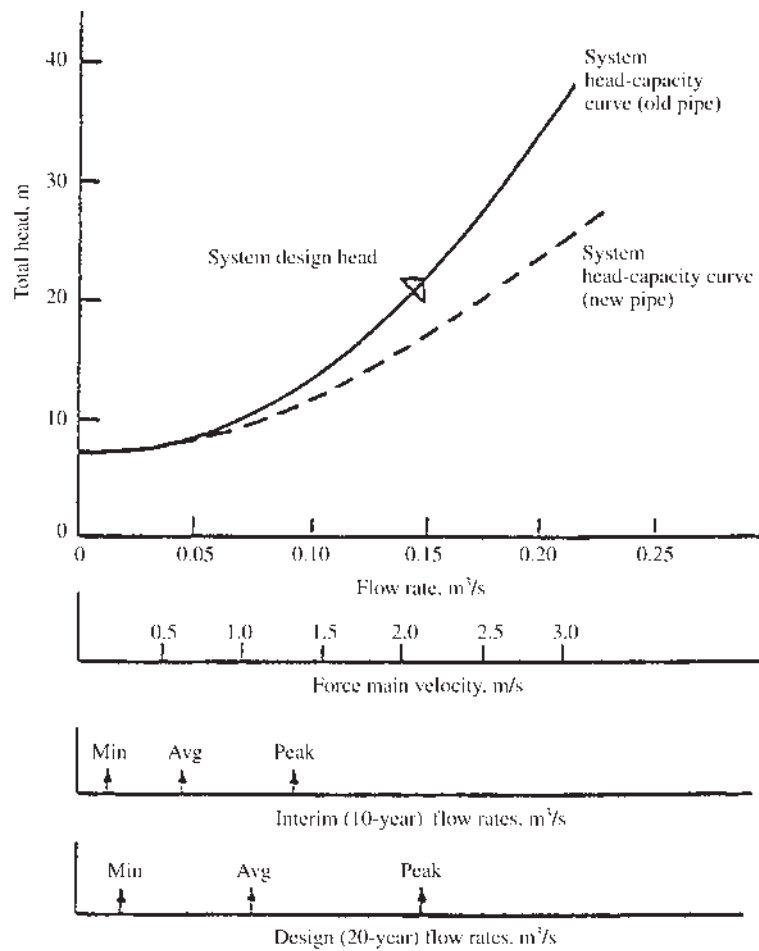


Figure 2.15 Selection of pump type.



**Figure 2.16** Variations in demand and operating characteristics.

- Seasonal variations in demand.
- Water demand increases as the system population expands.
- Increasing pipe friction as the system ages.

For all operating conditions, it is necessary to maintain pump operation close to peak efficiency. This can be achieved using multiple pumps and timed pumping periods.

## 2.14 CHANGING PUMP SPEED

The most common pump–motor combination employed in water supply operations is a close coupled system powered by an electric motor. These units can only operate at a speed related to the frequency of the A.C. supply (50 cycles/s or 3000 cycles/min), with the number of pairs of poles in the motor ( $N$ ) reducing the pump speed to  $3000/N$  revolutions per minute.

Pumps driven through belt drives or powered by petrol or diesel motors are more flexible allowing the pump speed to be adjusted to suit the operational requirements. Analysis of system operation will require the head–discharge–efficiency characteristic for the particular operating speed.

Given the head–discharge–efficiency characteristics for speed  $N$  (in tabular form), the corresponding characteristics for any speed  $N'$  can be established as follows:

$$Q' = \left(\frac{N'}{N}\right)Q \quad \text{flow points} \quad (2.20)$$

$$H' = \left(\frac{N'}{N}\right)^2 H \quad \text{head points} \quad (2.21)$$

$$\eta' = \eta \quad \text{efficiency points} \quad (2.22)$$

The data set for the new pump speed can then be matched to the system characteristics.

## 2.15 MULTIPLE PUMP OPERATION

The most common type of pumping station has two or more pumps operating in parallel. This provides flexibility of operation in coping with a range of flow conditions and allows maintenance of individual units while other units are in operation.

Occasionally situations will be encountered where pumps are operated in series to boost outputs. This is generally a temporary measure as any failure of one unit will severely affect system operation.

Composite characteristics (head–discharge curves) are obtained by combining the individual curves. The composite curve is employed in the same manner (i.e., intersection with system curve) as an individual curve (Fig. 2.17).

Where pumps operate in parallel, the composite curve is obtained by adding the *flow rates* for a given *head*.

Where pumps operate in series, the composite is obtained by adding *heads* for a given *flow rate*.

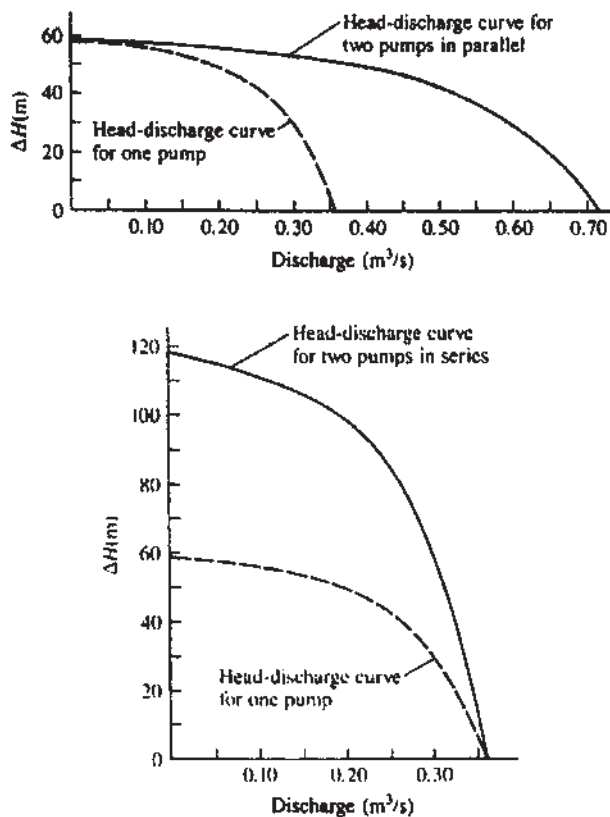
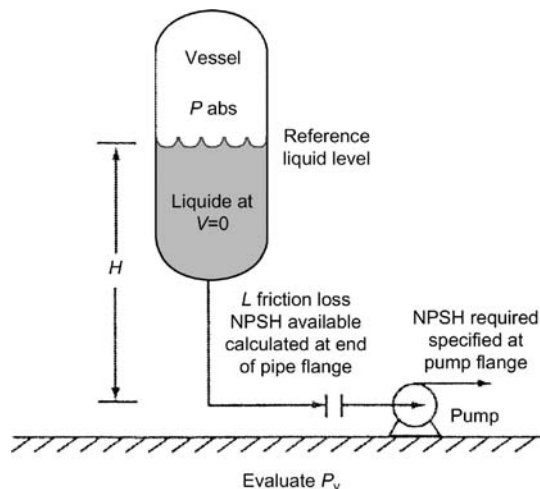


Figure 2.17 Composite pump characteristics.

### 2.15.1 Net Positive Suction Head

Simply, NPSH is the minimum suction condition (pressure) required to prevent pump cavitation. Conceptually, NPSH can be imagined as the pressure drop between the pump inlet flange and the point inside the pump where the fluid dynamic action, as it leaves the impeller, causes a pressure rise. Sufficient NPSH allows for pumping without liquid vaporizing in the pump first-stage impeller eye as the fluid pressure drops due to pump suction losses (Fig. 2.18).

The NPSH required is reported in head of fluid (being pumped) required at the pump inlet. As such, NPSH required has units of length (height). Usually, the datum line for pump NPSH is the centerline of the inlet. This is satisfactory for small pumps. For larger pumps, the NPSH requirements reported by the manufacturer should be verified for the datum line being discussed. The NPSH



**Figure 2.18** The elements of Eq. (2.24) are illustrated with a pump taking suction from a tower.

available differs from NPSH required. The NPSH required determined during the manufacturers test and shown on the vendor's pump curve is based upon a 3% head pump differential loss. The NPSH available must be large enough to eliminate head loss. The NPSH available is the excess of pressure over the liquid's vapor pressure at the pump suction flange. Except in rare circumstances, centrifugal pumps require the NPSH available to be greater than NPSH required to avoid cavitation during operation. Determining the NPSH available is the process engineer's job. Determining the NPSH required is the job of the pump vendor.

Our concern here is the process system side of determining what NPSH is available. Pressure balance and NPSH available derive from Bernoulli's equation for a stationary conduit where the total energy (head) in a system is the same for any point along a streamline (assuming no friction losses). Incorporating friction losses and restating the formula in a form familiar to process engineers, the NPSH available in a system can be expressed as:

$$\text{NPSH}_a = \frac{2.31(P + P_a - P_v)}{\gamma} + \left( S - B - L + \frac{V^2}{2g} \right) \quad (2.23)$$

where  $\text{NPSH}_a$  is the net positive suction head available (ft);  $P$ , pressure above liquid (psi gage);  $P_a$ , atmospheric pressure (psi);  $P_v$ , vapor pressure of liquid at pumping conditions (psia);  $\gamma$ , specific gravity of liquid at pumping conditions;

$S$ , static height of liquid from grade (ft);  $B$ , distance of pump centerline (suction nozzle centerline for vertical pumps);  $L$ , suction system friction losses (ft of pumping liquid); and  $V$  is the average liquid velocity at pump suction nozzle (ft/s).

Converting to absolute pressures, fluid density and resetting the datum line to the pump centerline results in:

$$\text{NPSH}_a = \frac{144(P_{\text{abs}} - P_v)}{\rho} + H - L + \frac{V^2}{2g} \quad (2.24)$$

where  $P_{\text{abs}}$  is the pressure above liquids (psia);  $\rho$ , fluid density (lb/ft<sup>3</sup>); and  $H$  is the static height of liquid between liquid level and pump suction centerline (datum line), ft.

**Illustrative Example 2.1:** A centrifugal pump runs at a tip speed of 12 m/s and a flow velocity of 1.5 m/s. The impeller diameter is 1.2 m and delivers 3.8 m<sup>3</sup>/min of water. The outlet blade angle is 28° to the tangent at the impeller periphery. Assuming that the fluid enters in the axial direction and zero slip, calculate the torque delivered by the impeller.

**Solution:**

From Fig. 2.2, for zero slip  $\beta_2 = \beta_2^1$ . Using Eq. (2.1), the Euler head  $H = E = (U_2 C_{w2} - U_1 C_{w1})/g$ . Since  $C_{w1} = 0$ , as there is no inlet whirl component, head  $H$  is given by

$$\begin{aligned} H &= \frac{U_2 C_{w2}}{g} = \frac{U_2}{g} \left( U_2 - \frac{1.5}{\tan 28^\circ} \right) = \frac{12}{9.81} \left( 12 - \frac{1.5}{\tan 28^\circ} \right) \\ &= 11.23 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Power delivered} &= \rho g Q H \frac{J}{s} = \frac{1000(9.81)(3.8)(11.23)}{60(1000)} \\ &= 6.98 \text{ kW} \end{aligned}$$

$$\text{Torque delivered} = \text{Power/angular velocity} = 6980(0.6)/12 = 349 \text{ Nm.}$$

**Illustrative Example 2.2:** A fluid passes through an impeller of 0.22 m outlet diameter and 0.1 m inlet diameter. The impeller is rotating at 1250 rpm, and the outlet vane angle is set back at an angle of 22° to the tangent. Assuming that the fluid enters radially with velocity of flow as 3.5 m/s, calculate the head imparted to a fluid.

**Solution:**

Since fluid enters in the radial direction,  $C_{w1} = 0$ ,  $\alpha_1 = 90^\circ$ ,  $\beta_2 = 22^\circ$ ,  $C_{a1} = 3.5 \text{ m/s} = C_{a2}$

Head developed  $H = C_{w2}U_2/g$

Impeller tip speed,  $U_2 = \frac{\pi DN}{60} = \frac{\pi(0.22)(1250)}{60} = 14.40 \text{ m/s}$

Whirl velocity at impeller outlet, from velocity diagram,

$$\begin{aligned}C_{w2} &= U_2 - (C_{a2}/\tan \beta_2) \\ &= 14.40 - (3.5/\tan 22^\circ) = 5.74 \text{ m/s}\end{aligned}$$

Therefore, the head imparted is given by

$$H = 5.74(14.40)/9.81 = 8.43 \text{ m}$$

**Design Example 2.3:** A centrifugal pump impeller runs at 1400 rpm, and vanes angle at exit is  $25^\circ$ . The impeller has an external diameter of 0.4 m and an internal diameter of 0.2 m. Assuming a constant radial flow through the impeller at 2.6 m/s, calculate (1) the angle made by the absolute velocity of water at exit with the tangent, (2) the inlet vane angle, and (3) the work done per kg of water (Fig. 2.19).

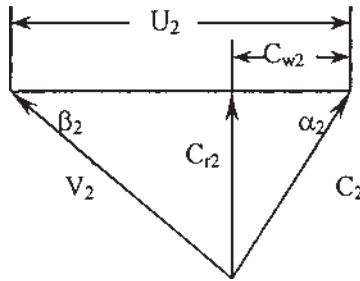
**Solution:**

1. Impeller tip speed is given by

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.4)(1400)}{60} = 29.33 \text{ m/s}$$

Whirl velocity at impeller tip

$$C_{w2} = U_2 - \frac{C_{r2}}{\tan \beta_2} = 29.33 - \frac{2.6}{\tan 25^\circ} = 23.75 \text{ m/s}$$



**Figure 2.19** Velocity triangle at outlet.

Now, from the velocity triangle at impeller tip,

$$\tan \alpha_2 = \frac{C_{r2}}{C_{w2}} = \frac{2.6}{23.75} = 0.1095$$

Therefore,  $\alpha_2 = 6.25^\circ$ .

2. Impeller velocity at inlet

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi(0.2)1400}{60} = 14.67 \text{ m/s}$$

$$\tan \beta_1 = \frac{C_{r1}}{U_1} = \frac{2.6}{14.67} = 0.177$$

Therefore,  $\beta_1 = 10.05^\circ$ .

3. Work done per kg of water is given by

$$C_{w2}U_2 = 23.75(29.33) = 696.59 \text{ Nm} = 696.59 \text{ J.}$$

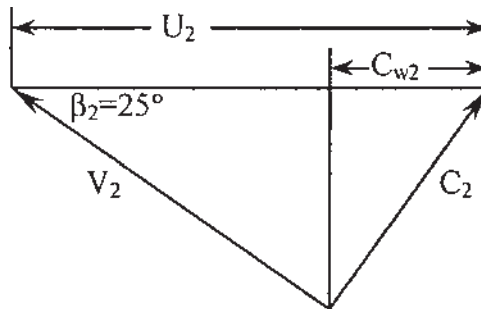
**Design Example 2.4:** A centrifugal pump impeller has a diameter of 1.2 m; rpm 210; area at the outer periphery  $0.65 \text{ m}^2$ ; angle of vane at outlet  $25^\circ$ , and ratio of external to internal diameter 2:1. Calculate (1) the hydraulic efficiency, (2) power, and (3) minimum speed to lift water against a head of 6.2 m. Assume that the pump discharges 1550 l/s (Fig. 2.20).

**Solution:**

1. Here,  $Q = 1550 \text{ l/s}$ ,  $\beta_2 = 25^\circ$ ,  $H = 6.2 \text{ m}$ ,  $D_2/D_1 = 2$ ,  $D_2 = 1.2 \text{ m}$ ,  $N = 210 \text{ rpm}$ ,  $A = 0.65 \text{ m}^2$ .

Velocity of flow at impeller tip

$$C_{r2} = \frac{Q}{A} = \frac{1550}{1000(0.65)} = 2.385 \text{ m/s}$$



**Figure 2.20** Velocity triangle at impeller outlet.



Impeller tip speed,

$$U_2 = \frac{\pi(1.2)(210)}{60} = 13.2 \text{ m/s}$$

$$C_{w2} = U_2 - \frac{C_{r2}}{\tan \beta_2} = 13.2 - \frac{2.385}{\tan 25^\circ} = 8.09 \text{ m/s}$$

Assuming radial entry, theoretical head is given by

$$H = \frac{C_{w2}U_2}{9.81} = \frac{8.09(13.2)}{9.81} = 10.89 \text{ m}$$

Assuming slip factor,  $\sigma = 1$ , hydraulic efficiency is given by

$$\eta_h = \frac{6.2(100)}{10.89} = 56.9\%$$

2. Power  $P = (1550)(10.89)(9.81)/1000 = 165.59 \text{ kW}$ .

3. Centrifugal head is the minimum head. Therefore,

$$\frac{U_2^2 - U_1^2}{2g} = 6.2$$

It is given that  $U_1 = U_2/2$ . Therefore,

$$U_2^2 - 0.25U_2^2 = 2(9.81)(6.2)$$

i.e.,  $U_2 = 12.74 \text{ m/s}$

Hence, minimum speed is  $= 12.74(60)/\pi(1.2) = 203 \text{ rpm}$ .

**Illustrative Example 2.5:** A centrifugal pump is required to pump water against a total head of 35 m at the rate of 45 l/s. Find the horsepower of the pump, if the overall efficiency is 60%.

**Solution:**

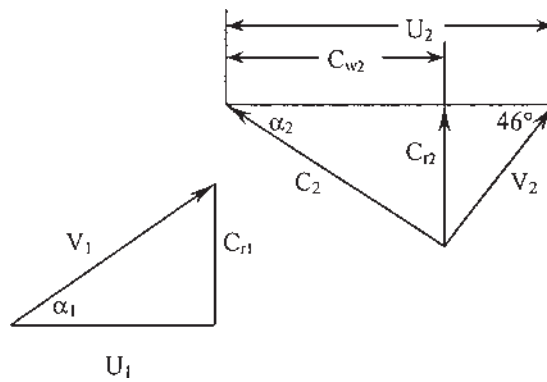
Total head,  $H = 35 \text{ m}$

Discharge,  $Q = 45 \text{ l/s} = 0.045 \text{ m}^3/\text{s}$

Overall efficiency,  $\eta_o = 60\% = 0.60$

$$\begin{aligned} \text{Power, } P &= \frac{\rho g Q H J}{\eta_o s} = \frac{\rho g Q H}{1000 \eta_o} \text{ kW} \\ &= \frac{9.81(0.045)(35)}{0.746(0.60)} \\ &= 34.5 \text{ hp} \end{aligned}$$

**Illustrative Example 2.6:** A centrifugal pump impeller has 0.3 m inlet diameter and 0.6 m external diameters. The pump runs at 950 rpm, and the entry



**Figure 2.21** Velocity triangle at impeller outlet and inlet.

of the pump is radial. The velocity of flow through the impeller is constant at 3.5 m/s. The impeller vanes are set back at angle of  $46^\circ$  to the outer rim. Calculate (1) the vane angle at inlet of a pump, (2) the velocity direction of water at outlet, and (3) the work done by the water per kg of water (Fig. 2.21).

**Solution:**

1. Velocity of flow,  $C_{r1} = C_{r2} = 3.5$  m/s. Let  $\alpha_1$  be the vane angle at inlet. Tangential velocity of impeller at inlet

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi(0.3)(950)}{60} = 14.93 \text{ m/s}$$

From inlet velocity triangle

$$\tan \alpha_1 = \frac{C_{r1}}{U_1} = \frac{3.5}{14.93} = 0.234$$

Therefore,  $\alpha_1 = 13.19^\circ$ .

2. Tangential velocity of impeller at outlet

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.6)(950)}{60} = 29.86 \text{ m/s}$$

For velocity of whirl at impeller outlet, using velocity triangle at outlet

$$C_{w2} = U_2 - \frac{C_{r2}}{\tan 46^\circ} = 29.86 - \frac{3.5}{\tan 46^\circ} = 26.48 \text{ m/s}$$

$$\text{and } C_2^2 = C_{r2}^2 + C_{w2}^2 = 3.5^2 + 26.48^2, \quad C_2 = 26.71 \text{ m/s}$$

where  $C_2$  is the velocity of water at outlet. Let  $\alpha_2$  be the direction of

water at outlet, and, therefore,  $\alpha_2$  is given by

$$\tan \alpha_2 = \frac{C_{r2}}{C_{w2}} = \frac{3.5}{26.48} = 0.132$$

i.e.,  $\alpha_2 = 7.53^\circ$ .

3. Work done by the wheel per kg of water

$$W = C_{w2}U_2 = 26.48(29.86) = 790.69 \text{ Nm}$$

**Design Example 2.7:** A centrifugal pump delivers water at the rate of  $8.5 \text{ m}^3/\text{min}$  against a head of 10 m. It has an impeller of 50 cm outer diameter and 25 cm inner diameter. Vanes are set back at outlet at an angle of  $45^\circ$ , and impeller is running at 500 rpm. The constant velocity of flow is 2 m/s. Determine (1) the manometric efficiency, (2) vane angle at inlet, and (3) minimum starting speed of the pump (Fig. 2.22).

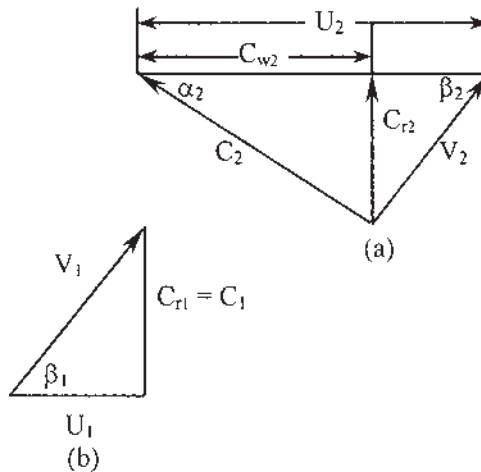
**Solution:**

1. The manometric efficiency is given by

$$\eta_{\text{man}} = \frac{H}{(C_{w2}U_2/g)}$$

From outlet velocity triangle

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.5)(500)}{60} = 13.0 \text{ m/s}$$



**Figure 2.22** Velocity triangle (a) outlet, (b) inlet.

Now,  $\tan \beta_2 = C_{r2}/(U_2 - C_{w2})$ , or  $\tan 45^\circ = 2/(U_2 - C_{w2})$ , or  $1 = 2/(13 - C_{w2})$ ,  $C_{w2} = 11$  m/s.

$$\text{Hence, } \eta_{\text{man}} = \frac{H}{(C_{w2}U_2/g)} = \frac{10(9.81)}{11(13)} = 68.6\%$$

2. Vane angle at inlet  $\beta_1$

$$\tan \beta_1 = \frac{C_{r1}}{U_1} \text{ and } U_1 = 0.5 \times U_2 = 6.5 \text{ m/s}$$

$$\therefore \tan \beta_1 = \frac{2}{6.5} = 0.308$$

i.e.,  $\beta_1 = 17^\circ$ .

3. The minimum starting speed is

$$(U_2^2 - U_1^2)/2g = H \text{ or } \frac{(\frac{\pi D_2 N}{60})^2 - (\frac{\pi D_1 N}{60})^2}{2(9.81)} = 10$$

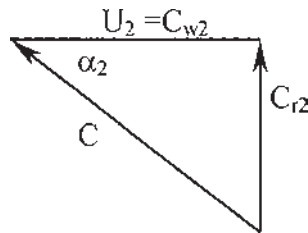
Therefore,  $N = 618$  rpm.

**Illustrative Example 2.8:** A centrifugal pump impeller has 0.6 m outside diameter and rotated at 550 rpm. Vanes are radial at exit and 8.2 cm wide. Velocity of flow through the impeller is 3.5 m/s, and velocity in the delivery pipe is 2.5 m/s. Neglecting other losses, calculate head and power of the pump (Fig. 2.23).

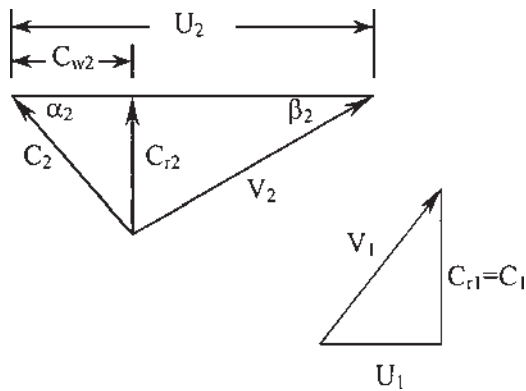
**Solution:**

1.  $D_2 = 0.6$  m,  $N = 550$  rpm,  $C_{r2} = 3.5$  m/s,  $C_{w2} = U_2$ .  
Impeller speed at outlet

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.6)(550)}{60} = 17.29 \text{ m/s.}$$



**Figure 2.23** Velocity triangle for Example 2.8.



**Figure 2.24** Velocity triangles for Example 2.9.

Head, through which the water can be lifted,

$$\begin{aligned}
 H &= \frac{C_{w2}U_2}{g} - \frac{V_2^2}{2g} \text{ (neglecting all losses)} \\
 &= \frac{(17.29)(17.29)}{9.81} - \frac{2.5^2}{2(9.81)} = 30.47 - 0.319 \\
 &= 30.2 \text{ m of water.}
 \end{aligned}$$

$$\begin{aligned}
 2. \text{ Power} &= \frac{\rho g Q H}{1000} \text{ kW} \\
 \text{where } Q &= \pi D_2 b_2 C_{r2} \text{ (where } b_2 \text{ is width)} \\
 &= \pi(0.6)(0.082)(3.5) = 0.54 \text{ m}^3/\text{s}
 \end{aligned}$$

Therefore, power is given by

$$P = \frac{\rho g Q H}{1000} \text{ kW} = \frac{1000(9.81)(0.54)(30.2)}{1000} = 160 \text{ kW.}$$

**Illustrative Example 2.9:** A centrifugal pump impeller has a diameter of 1 m and speed of 11 m/s. Water enters radially and discharges with a velocity whose radial component is 2.5 m/s. Backward vanes make an angle of  $32^\circ$  at exit. If the discharge through the pump is  $5.5 \text{ m}^3/\text{min}$ , determine (1) h.p. of the pump and (2) turning moment of the shaft (Fig. 2.24).

**Solution:**

1. Data

$$D_2 = 1 \text{ m,}$$

$$U_2 = 11 \text{ m/s,}$$

$$\alpha_1 = 90^\circ,$$

$$C_{r2} = 2.5 \text{ m/s},$$

$$\beta_2 = 32^\circ,$$

$$Q = 5.5 \text{ m}^3/\text{min}.$$

First, consider outlet velocity triangle

$$\begin{aligned} C_{w2} &= U_2 - \frac{C_{r2}}{\tan \beta_2} \\ &= 11 - \frac{2.5}{\tan 32^\circ} = 7 \text{ m/s} \end{aligned}$$

Power of the pump is given by

$$P = \frac{\rho Q C_{w2} U_2}{1000} \text{ kW}$$

$$P = \frac{(1000)(5.5)(7)(11)}{(60)(1000)} = 7 \text{ kW}.$$

$$2. \text{ Now, h.p.} = \frac{2\pi NT}{60}$$

where  $T$  is the torque of the shaft. Therefore,

$$T = \frac{\text{h.p.} \times 60}{2\pi N}$$

$$\text{But } U_2 = \frac{\pi D_2 N}{60} \text{ or } N = \frac{60 \times U_2}{\pi D_2} = \frac{60(11)}{\pi(1)} = 210 \text{ rpm}$$

Hence,

$$T = \frac{(7)(1000)(60)}{2\pi(210)} = 318 \text{ Nm/s}.$$

**Illustrative Example 2.10:** A centrifugal pump running at 590 rpm and discharges 110 l/min against a head of 16 m. What head will be developed and quantity of water delivered when the pump runs at 390 rpm?

**Solution:**

$$N_1 = 590,$$

$$Q_1 = 110 \text{ l/min} = 1.83 \text{ l/s},$$

$$H_1 = 16 \text{ m},$$

$$N_2 = 390 \text{ rpm},$$

$$H_2 = ?$$

$$\text{As } \frac{\sqrt{H_1}}{N_1} = \frac{\sqrt{H_2}}{N_2}$$

$$\text{Then, } \frac{\sqrt{16}}{590} = \frac{\sqrt{H_2}}{390}$$

Therefore,  $H_2 = 6.98 \text{ m}$

Therefore, head developed by the pump at 390 rpm = 6.98 m. In order to find discharge through the pump at 390 rpm, using the Eq. (1.11)

$$\frac{N_1\sqrt{Q_1}}{H_1^{3/4}} = \frac{N_2\sqrt{Q_2}}{H_2^{3/4}}$$

$$\frac{590\sqrt{1.83}}{(16)^{3/4}} = \frac{390\sqrt{Q_2}}{(6.98)^{3/4}} \quad \text{or} \quad \frac{798.14}{8} = \frac{390\sqrt{Q_2}}{4.29}$$

$$\sqrt{Q_2} = 1.097$$

i.e.,  $Q = 1.203 \text{ l/s}$

**Illustrative Example 2.11:** The impeller of a centrifugal pump has outlet diameter of 0.370 m, runs at 800 rpm, and delivers 30 l/s of water. The radial velocity at the impeller exit is 2.5 m/s. The difference between the water levels at the overhead tank and the pump is 14 m. The power required to drive the pump is 8 hp, its mechanical and volumetric effectiveness being 0.96 and 0.97, respectively. The impeller vanes are backward curved with an exit angle of  $45^\circ$ . Calculate (1) ideal head developed with no slip and no hydraulic losses and (2) the hydraulic efficiency.

**Solution:**

1. Impeller tip speed

$$U_2 = \frac{\pi D_2 N}{60}$$

or

$$U_2 = \frac{\pi(0.37)(800)}{60} = 15.5 \text{ m/s.}$$

As the radial velocity at the impeller exit = 2.5 m/s.

Therefore,  $C_{w2} = U_2 - \frac{C_{r2}}{\tan \beta_2} = 15.5 - \frac{2.5}{\tan 45^\circ} = 13 \text{ m/s.}$

When there is no slip, the head developed will be

$$H = \frac{C_{w2} U_2}{g} = \frac{(13)(15.5)}{9.81} = 20.54 \text{ m}$$

If there are no hydraulic internal losses, the power utilized by the pump will be:

$$P = (0.96)(8) = 7.68 \text{ hp}$$

$$\text{Theoretical flow rate} = \frac{Q}{\eta_v} = \frac{0.03}{0.97} = 0.031 \text{ m}^3/\text{s}$$

Ideal head,  $H_i$ , is given by

$$H_i = \frac{(7.68)(0.746)}{(9.81)(0.031)} = 18.84 \text{ m.}$$

2. The hydraulic efficiency is

$$\eta_h = \frac{H}{H_i} = \frac{14}{18.84} = 0.746 \quad \text{or} \quad 74.3\%.$$

**Illustrative Example 2.12:** The impeller of a centrifugal pump has outer diameter of 1.06 m and speed is 56 m/s. The blades are backward curved and they make an angle of  $20^\circ$  with the wheel tangent at the blade tip. If the radial velocity of the flow at the tip is 7.5 m/s and the slip factor is 0.88. Determine (1) the actual work input per kg of water flow and (2) the absolute velocity of fluid at the impeller.

**Solution:**

1. Exit blade angle,  $\beta_2 = 20^\circ$

$$\therefore C_{w2} = U_2 - \frac{C_{r2}}{\tan \beta_2} = 56 - \frac{7.5}{\tan 20^\circ} = 35.4 \text{ m/s}$$

Using slip factor,  $\sigma = 0.88$ , the velocity whirl at exit is,  $C_{w2} = \sigma \times 35.4 = 0.88 \times 35.4 = 31.2 \text{ m/s}$ .

Work input per kg of water flow

$$W = \frac{C_{w2}U_2}{1000} = \frac{(56)(31.2)}{1000} = 1.75 \text{ kJ/kg.}$$

2. Absolute velocity at impeller tip

$$C_2 = \sqrt{(C_{r2}^2 + C_{w2}^2)} = \sqrt{(7.5^2 + 31.2^2)}$$

$$C_2 = \sqrt{56.25 + 973.44} = 32.09 \text{ m/s}$$

**Design Example 2.13:** A centrifugal pump impeller of 260 mm diameter runs at 1400 rpm, and delivers  $0.03 \text{ m}^3/\text{s}$  of water. The impeller has



a backward curved facing blades inclined at  $30^\circ$  to the tangent at outlet. The blades are 20 mm in depth at the outlet, and a slip factor of 0.78 may be assumed. Calculate the theoretical head developed by the impeller, and the number of impeller blades.

**Solution:**

Assuming the blades are of infinitesimal thickness, the flow area is given by

$$\begin{aligned} A &= \text{impeller periphery} \times \text{blade depth} \\ &= \pi \times 0.26 \times 0.02 = 0.0163 \text{ m}^2 \end{aligned}$$

Flow velocity is given by

$$C_{r2} = \frac{Q}{A} = \frac{0.03}{0.0163} = 1.84 \text{ m/s}$$

Impeller tip speed,  $U_2$ , is

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.26)(1400)}{60} = 19.07 \text{ m/s}$$

Absolute whirl component,  $C_{w2}$  is given by

$$C_{w2} = U_2 - \frac{C_{r2}}{\tan 30^\circ} = 19.07 - \frac{1.84}{\tan 30^\circ} = 15.88 \text{ m/s}$$

Using Euler's equation, and assuming  $C_{w1} = 0$  (i.e., no whirl at inlet)

$$H = \frac{U_2 C_{w2}}{g} = \frac{(19.07)(15.88)}{9.81} = 30.87 \text{ m}$$

Theoretical head with slip is  $H = 0.78 \times 30.87 = 24.08 \text{ m}$ .

To find numbers of impeller blades, using Stanitz formula

$$\text{Slip factor, } \sigma = 1 - \frac{0.63\pi}{n} \text{ or } 0.78 = 1 - \frac{0.63\pi}{n} \text{ or } \frac{0.63\pi}{n} = 1 - 0.78 = 0.22$$

$$\therefore n = \frac{0.63\pi}{0.22} = 9$$

Number of blades required = 9

**Design Example 2.14:** A centrifugal pump impeller runs at 1500 rpm, and has internal and external diameter of 0.20 m and 0.4 m, respectively, assuming constant radial velocity at 2.8 m/s and the vanes at the exit are set back at an angle of  $30^\circ$ . Calculate (1) the angle absolute velocity of water at exit makes with the tangent, (2) inlet vane angle, and (3) the work done per kg of water.

**Solution:**

1.  $D_1 = 0.2$  m,  $D_2 = 0.4$  m,  $N = 1500$  rpm,  $C_{r2} = 2.8$  m/s,  $\beta_2 = 30^\circ$ .  
Impeller tip speed,  $U_2$ , is

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.4)(1500)}{60} = 31.43 \text{ m/s}$$

Whirl component of absolute velocity at impeller exit is

$$C_{w2} = U_2 - \frac{C_{r2}}{\tan 30^\circ} = 31.43 - \frac{2.8}{\tan 30^\circ} = 26.58 \text{ m/s}$$

$$\tan \alpha_2 = \frac{2.8}{26.58} = 0.1053$$

i.e.,  $\alpha_2 = 6^\circ$ .

2. Impeller speed at inlet

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi(0.2)(1500)}{60} = 15.7 \text{ m/s}$$

$$\tan \beta_1 = \frac{2.8}{15.7} = 0.178$$

i.e.,  $\beta_1 = 10.1^\circ$ .

3. Work done per kg of water

$$C_{w2}U_2 = 26.58 \times 31.43 = 835.4 \text{ Nm.}$$

**Design Example 2.15:** An axial flow pump discharges water at the rate of  $1.30 \text{ m}^3/\text{s}$  and runs at 550 rpm. The total head is 10 m. Assume blade velocity = 22 m/s, the flow velocity = 4.5 m/s, hydraulic efficiency = 0.87, and the overall pump efficiency = 0.83, find (1) the power delivered to the water, and power input, (2) the impeller hub diameter and tip diameter, and (3) the inlet and outlet blade angles for the rotor.

**Solution:**

1. Power delivered to the water

$$\begin{aligned} P &= \rho g H Q / 1000 \text{ kW} \\ &= (9.81)(1.30)(10) = 127.53 \text{ kW} \end{aligned}$$

Power input to the pump

$$P = \frac{127.53}{0.83} = 153.65 \text{ kW.}$$

2. Rotor tip diameter is given by

$$D_2 = \frac{60U_2}{\pi N} = \frac{(60)(22)}{\pi(550)} = 0.764 \text{ m}$$

Rotor hub diameter

$$D_1^2 = D_2^2 - \frac{Q}{(\pi/4) \times C_a} = 0.764^2 - \frac{1.3}{(\pi/4)(4.5)} = 0.216 \text{ m}$$

i.e.,  $D_1 = 0.465 \text{ m}$ .

3. Rotor velocity at hub is given by

$$U_1 = \frac{D_1}{D_2} U_2 = \frac{(0.465)(22)}{0.764} = 13.39 \text{ m/s}$$

Since, the axial velocity is constant, we have: rotor inlet angle at tip

$$\alpha_{1t} = \tan^{-1}(C_a/U_1) = \tan^{-1}(4.5/13.39) = 18.58^\circ$$

Rotor outlet angle

$$\alpha_{2t} = \tan^{-1}(C_a/U_2) = \tan^{-1}(4.5/22) = 11.56^\circ.$$

**Design Example 2.16:** A single stage, radial flow, and double suction centrifugal pump having the following data:

Discharge	72 l/s
Inner diameter	90 mm
Outer diameter	280 mm
Revolution/minute	1650
Head	25 m
Width at inlet	20 mm/side
Width at outlet	18 mm/side
Absolute velocity angle at inlet	90°
Leakage losses	2 l/s
Mechanical losses	1.41 kW
Contraction factor due to vane thickness	0.85
Relative velocity angle measured from tangential direction	35°
Overall efficiency of the pump	0.56

Determine (1) inlet vane angle, (2) the angle at which the water leaves the wheel, (3) the absolute velocity of water leaving impeller, (4) manometric efficiency, and (5) the volumetric and mechanical efficiencies.

**Solution:**

Total quantity of water to be handled by the pump

$$\begin{aligned} Q_t &= Q_{\text{del}} + Q_{\text{leak}} \\ &= 72 + 2 = 74 \end{aligned}$$

Total quantity of water per side =  $74/2 = 37$  l/s

1. Impeller speed at inlet

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi(0.09)(1650)}{60} = 7.78 \text{ m/s}$$

$$\begin{aligned} \text{Flow area at inlet} &= \Pi D_1 b_1 \times \text{contraction factor} \\ &= (\Pi)(0.09)(0.02)(0.85) = 0.0048 \text{ m}^2 \end{aligned}$$

Therefore, the velocity of flow at inlet

$$C_{r1} = \frac{Q}{\text{Area of flow}} = \frac{37 \times 10^{-3}}{0.0048} = 7.708 \text{ m/s}$$

From inlet velocity triangle

$$\tan \beta_1 = \frac{C_{r1}}{U_1} = \frac{7.708}{7.78} = 0.9907$$

$$\beta_1 = 44.73^\circ$$

2. Area of flow at outlet

$$A_2 = \Pi \times D_2 \times b_2 \times \text{contraction factor}$$

Where  $b_2 = 18/2 = 9$  mm for one side.

$$\text{So, } A_2 = (\Pi)(0.28)(0.009)(0.85) = 0.0067 \text{ m}^2.$$

Therefore, the velocity of flow at outlet

$$C_{r2} = \frac{Q}{\text{Area of flow}} = \frac{37 \times 10^{-3}}{0.0067} = 5.522 \text{ m/s}$$

The impeller speed at outlet

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi(0.28)(1650)}{60} = 24.2 \text{ m/s}$$

Now using velocity triangle at outlet

$$\tan \beta_2 = \frac{C_{r2}}{U_2 - C_{w2}} = \frac{5.522}{24.2 - C_{w2}}$$

$$C_{w2} = 16.99 \text{ m/s}$$

Further,

$$\tan \alpha_2 = \frac{C_{r2}}{C_{w2}} = \frac{5.522}{16.99} = 0.325$$

$$\alpha_2 = 18^\circ.$$

3. The absolute velocity of water leaving the impeller

$$C_2 = \frac{C_{w2}}{\cos \alpha_2} = \frac{16.99}{\cos 18^\circ} = 17.8 \text{ m/s.}$$

4. The manometric efficiency

$$\eta_{\text{mano}} = \frac{(g)(H_{\text{mano}})}{(U_2)(C_{w2})} = \frac{9.81 \times 25}{24.2 \times 16.99} = 0.596.$$

5. The volumetric efficiency

$$\eta_v = \frac{Q_2}{Q_{\text{Total}}} = \frac{72}{74} = 0.973$$

$$\text{Water power} = \rho g Q H$$

$$= 1000 \times 9.81 \times 72 \times 25 / 1000$$

$$= 17.66 \text{ kW}$$

$$\text{Shaft power} = \frac{\text{Water power}}{\eta_o} = \frac{17.66}{0.56} = 31.54 \text{ kW}$$

Mechanical efficiency is

$$\eta_m = \frac{P_s - P_{\text{loss}}}{P_s} = \frac{31.54 - 1.41}{31.54} = 0.955 \quad \text{or} \quad 95.5\%.$$

**Illustrative Example 2.17:** A single stage centrifugal pump is designed to give a discharge of  $Q$  when working against a manometric head of 20 m. On test, it was found that head actually generated was 21.5 m for the designed discharge,  $Q$ . If it is required to reduce the original diameter 32 cm without reducing the speed of the impeller, compute the required diameter to be reduced.

**Solution:**

Head generated by the pump

$$H = \frac{U^2}{2g} = \frac{(\pi DN/60)^2}{2g}$$

or

$$H \propto D^2$$

$$\frac{H}{H'} = \left(\frac{D}{D'}\right)^2$$

$$H = 21.5 \text{ m}, H' = 20 \text{ m}, D = 32 \text{ cm}$$

So,

$$D' = D \left(\frac{H'}{H}\right)^{1/2} = 32 \left(\frac{20}{21.5}\right)^2 = 30.86 \text{ cm}$$

**Design Example 2.18:** A two stage centrifugal pump is designed to discharge 55 l/s at a head of 70 m. If the overall efficiency is 76% and specific speed per stage about 38, find (1) the running speed in rpm and (2) the power required to run pump.

If the actual manometric head developed is 65% of the theoretical head, assuming no slip, the outlet angle of the blades 28°, and radial velocity at exit 0.14 times the impeller tip speed at exit, find the diameter of impeller.

**Solution:**

1. The specific speed is

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$N = \frac{N_s H^{3/4}}{\sqrt{Q}} = \frac{38(70/2)^{3/4}}{\sqrt{55 \times 10^{-3}}} = \frac{546.81}{0.235} = 2327 \text{ rpm.}$$

2.  $Q = 55 \times 10^{-3} \text{ m}^3/\text{s}$

Power required to drive pump

$$\begin{aligned} &= \frac{\rho g Q H}{0.76} = \frac{1000 \times 9.81 \times 55 \times 10^{-3} \times 70}{0.76 \times 1000} \\ &= 49.7 \text{ kW} \end{aligned}$$

$$H_{\text{mano}} = 0.65 H.$$

$$\text{Here } \beta_2 = 28^\circ \text{ and } C_{r2} = 0.14U_2.$$

From velocity triangle at outlet

$$\tan \beta_2 = \frac{C_{r2}}{U_2 - C_{w2}}$$

$$\text{or } \tan 28^\circ = \frac{0.14U_2}{U_2 - C_{w2}}$$

$$\frac{U_2}{U_2 - C_{w2}} = \frac{0.5317}{0.14} = 3.798 \quad (\text{A})$$

As the flow at entrance is radial and  $\alpha_1 = 90^\circ$ , the fundamental equation of pump would be

$$\frac{H_{\text{mano}}}{\eta_{\text{mano}}} = \frac{U_2 C_{w2}}{g}$$

Where  $\eta_{\text{mano}}$  manometric efficiency of pump which is 65%.

$$\text{Therefore, } \frac{35}{0.65} = \frac{U_2 C_{w2}}{g}$$

$$U_2 C_{w2} = \frac{35 \times 9.81}{0.65}$$

$$C_{w2} = \frac{528.23}{U_2} \quad (\text{B})$$

Substituting for  $C_{w2}$  in Eq. (A) and solving

$$\frac{U_2}{U_2 - \frac{528.23}{U_2}} = 3.798$$

$$U_2 = 26.78 \text{ m/s.}$$

Also,

$$U_2 = \frac{\pi D_2 N}{60}$$

$$\text{or } 26.78 = \frac{\pi \times D_2 \times 2327}{60}$$

$$D_2 = 0.2197 \text{ m or } 21.97 \text{ cm.}$$

**Design Example 2.19:** Two multistage centrifugal pumps are used in series to handle water flow rate of  $0.0352 \text{ m}^3/\text{s}$ , and total head required is 845 m. Each pump is required to produce a head of half the total and run at 1445 rpm. If the impeller in all the stages is identical and specific speed is 14, determine (1) head developed per stage and the required number of stages in each pump, (2) The required impeller diameters assuming the speed ratio based on the outer

tip diameter to be 0.96 and the shaft power input, if the overall efficiency of each pump is 0.75.

**Solution:**

Head developed in each stage is

$$H^{3/4} = \frac{N\sqrt{Q}}{N_s} = \frac{1445\sqrt{0.0352}}{14}$$

$$H = 51.93 \text{ m}$$

Total head required = 845 m (of water)

$$\text{Number of stages needed} = \frac{845}{51.93} = 16$$

Number of stages in each pump = 8

Impeller speed at tip is

$$\begin{aligned} U_2 &= 0.96(2gH)^{0.5} \\ &= 0.96[2 \times 9.81 \times 51.93]^{0.5} \\ &= 30.6 \text{ m/s} \end{aligned}$$

Impeller diameter at tip,  $D_2 = \pi \times 60 \times 30.6 \times 1445$ . But

$$U_2 = \frac{\pi D_2 N}{60}$$

or

$$D_2 = \frac{U_2 \times 60}{\pi \times 1445} = \frac{30.6 \times 60}{\pi \times 1445} = 0.4043 \text{ m} \quad \text{or} \quad 40.43 \text{ cm.}$$

**Design Example 2.20:** A centrifugal pump is required to be made to lift water through 105 m heights from a well. Number of identical pumps having their designed speed 900 rpm and specific speed 700 rpm with a rated discharge of 5500 l/min are available. Determine the number of pumps required and how they should be connected?

**Solution:**

Specific speed for a single impeller is given by

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

$$\text{Given, } N_s = 700, \quad H = 105 \quad N = 900, \quad \text{and} \quad Q = \frac{5500}{60} = 91.67 \text{ l/s}$$

Substituting,

$$700 = \frac{900\sqrt{91.67}}{H^{3/4}}, \quad H = 28 \text{ m}$$



Hence number of stages required

$$\begin{aligned} &= \frac{\text{Total head to be developed}}{\text{Head per stage}} \\ &= \frac{105}{28} = 4 \text{ stages in series} \end{aligned}$$

**Design Example 2.21:** The specific speed of an axial flow pump impeller is 1150 and velocity of flow is 2.5 m/s. The outer and inner diameters of the impeller are 0.90 m and 0.45 m, respectively. Calculate the suitable speed of the pump to give a head of 5.5 m. Also, calculate vane angle at the entry of the pump.

**Solution:**

Given,

$$D_2 = 0.9 \text{ m}, D_1 = 0.45 \text{ m}, N_s = 1150, C_r = 2.5 \text{ m/s}, H = 5.5 \text{ m}.$$

As discharge,  $Q = \text{area of flow} \times \text{velocity of flow}$

$$\begin{aligned} &= \frac{\pi}{4}(0.9^2 - 0.45^2) \times 2.5 = 1.193 \text{ m}^3/\text{s} \\ &= 1193 \text{ l/s} \end{aligned}$$

Also,

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$

or

$$1150 = \frac{N\sqrt{1193}}{(5.5)^{3/4}}$$

$$N = \frac{(5.5)^{3/4} \times 1150}{\sqrt{1193}} = 120 \text{ rpm}$$

In order to find vane angle at entry, using velocity triangle at inlet,

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.45 \times 120}{60} = 2.82 \text{ m/s}$$

$$\tan \alpha_1 \frac{C_{r1}}{U_1} = \frac{2.5}{2.82} = 0.8865$$

i.e.,

$$\alpha = 41.56^\circ.$$

## PROBLEMS

- 2.1** A centrifugal pump of 25 cm impeller diameter running at 1450 rpm, develops a head of 15 m. If the outlet flow area is  $480 \text{ cm}^2$ , and discharging water  $0.12 \text{ m}^3/\text{s}$ , and loss of head in the pump can be taken as  $0.003C_1^2$ , find the outlet blade angle.  
( $14^\circ$ )
- 2.2** A centrifugal pump having vane angles at inlet and outlet are  $25^\circ$  and  $30^\circ$ , respectively. If internal and external diameters of impeller are 0.15 and 0.30 m, respectively, calculate the work done per kg of water. Assume velocity of flow constant.  
(197.18 Nm)
- 2.3** A centrifugal pump discharges 50 liters/second of water against a total head of 40 m. Find the horsepower of the pump, if the overall efficiency is 62%.  
(42 hp)
- 2.4** A centrifugal pump delivers 26 l/s against a total head of 16 m at 1450 rpm. The impeller diameter is 0.5 m. A geometrically similar pump of 30 cm diameter is running at 2900 rpm. Calculate head and discharge required assuming equal efficiencies between the two pumps.  
(11.52 m, 11.23 l/s)
- 2.5** A centrifugal pump is built to work against a head of 20 m. A model of this pump built to one-fourth its size is found to generate a head of 7 m when running at its best speed of 450 rpm and requires 13.5 hp to run it. Find the speed of the prototype.  
(190 rpm)
- 2.6** Derive the expression for power required for a pump when it discharges a liquid of specific weight  $w$  at the rate of  $Q$  against a head of  $H$ .
- 2.7** Show that the pressure rise in an impeller of a centrifugal pump is given by  $\frac{C_{r1}^2 + U_2^2 - C_{r2}^2 \operatorname{cosec}^2 \beta_2}{2g}$  (where  $C_{r1}$  = velocity of flow at inlet,  $U_2$  = blade velocity at outlet,  $C_{r2}$  = velocity of flow at outlet, and  $\beta_2$  = blade angle at outlet). Assuming that friction and other losses are neglected.
- 2.8** Derive an expression for static head developed by a centrifugal pump having radial flow at inlet.
- 2.9** A centrifugal pump discharges  $0.15 \text{ m}^3/\text{s}$  of water against a head of 15 m. The impeller has outer and inner diameter of 35 and 15 cm, respectively. The outlet vanes are set back at an angle  $40^\circ$ . The area of flow is constant from inlet to outlet and is  $0.06 \text{ m}^2$ . Calculate the manometric efficiency

and vane angle at inlet if the speed of the pump is 960 rpm. Take slip factor = 1.

(57.3%, 18°)

- 2.10** A centrifugal pump of 35 cm diameter running at 1000 rpm develops a head of 18 m. The vanes are curved back at an angle of 30° to the tangent at outlet. If velocity flow is constant at 2.4 m/s, find the manometric efficiency of the pump.

(76.4%)

- 2.11** An axial flow pump is required to deliver 1 m<sup>3</sup>/s at 7 m head while running at 960 rpm. Its outer diameter is 50 and hub diameter is 25 cm. Find (1) flow velocity, which is assumed to be constant from hub to tip and (2) power required to drive the pump if overall efficiency is 84%.

(6.791 m/s, 81.75 kW)

- 2.12** An axial flow pump has the following data:

Rotational speed	750 rpm
Discharge of water	1.75 m <sup>3</sup> /s
Head	7.5 m
Hub to runner diameter ratio	0.45

Through flow velocity is 0.35 times the peripheral velocity. Find the diameter and minimum speed ratio.

(0.59 m, 0.83)

- 2.13** In an axial flow pump, the rotor has an outer diameter of 75 cm and an inner diameter of 40 cm; it rotates at 500 rpm. At the mean blade radius, the inlet blade angle is 12° and the outlet blade angle is 15°. Sketch the corresponding velocity diagrams at inlet and outlet, and estimate from them (1) the head the pump will generate, (2) the discharge or rate of flow in l/s, (3) the shaft h.p. input required to drive the pump, and (4) the specific speed of the pump. Assume a manometric or hydraulic efficiency of 88% and a gross or overall efficiency of 81%.

(19.8 m; 705 l/s; 230 hp; 45)

- 2.14** If an axial flow pump delivers a discharge  $Q$  against a head  $H$  when running at a speed  $N$ , deduce an expression for the speed of a geometrically similar pump of such a size that when working against unit head, it will transmit unit power to the water flowing through it. Show that this value is proportional to the specific speed of the pump.

## NOTATION

$b$	width of the diffuser passage
$C_{w2}$	tangential components of absolute velocity corresponding to the angle $\beta_2$
$E$	Euler head
$H$	total head developed by the pump
$H_i$	total head across the impeller
$N_{\text{suc}}$	Suction specific speed
$m$	mass flow rate
$n$	number of vanes
$P_s$	shaft power input
$Q$	flow rate
$r$	radius
$U$	impeller speed
$V$	relative velocity
$\alpha$	absolute velocity angle
$\beta$	relative velocity angle
$\eta_c$	casing efficiency
$\eta_R$	hydraulic efficiency
$\eta_i$	impeller efficiency
$\eta_m$	mechanical efficiency
$\eta_o$	overall efficiency
$\eta_v$	volumetric efficiency
$\rho$	density of liquid
$\sigma$	slip factor
$\omega$	angular velocity

## SUFFIXES

1	inlet to impeller
2	outlet from the impeller
3	outlet from the diffuser
a	axial
r	radial
w	whirl