7

Axial Flow and Radial Flow Gas Turbines

7.1 INTRODUCTION TO AXIAL FLOW TURBINES

The axial flow gas turbine is used in almost all applications of gas turbine power plant. Development of the axial flow gas turbine was hindered by the need to obtain both a high-enough flow rate and compression ratio from a compressor to maintain the air requirement for the combustion process and subsequent expansion of the exhaust gases. There are two basic types of turbines: the axial flow type and the radial or centrifugal flow type. The axial flow type has been used exclusively in aircraft gas turbine engines to date and will be discussed in detail in this chapter. Axial flow turbines are also normally employed in industrial and shipboard applications. Figure 7.1 shows a rotating assembly of the Rolls-Royce Nene engine, showing a typical single-stage turbine installation. On this particular engine, the single-stage turbine is directly connected to the main and cooling compressors. The axial flow turbine consists of one or more stages located immediately to the rear of the engine combustion chamber. The turbine extracts kinetic energy from the expanding gases as the gases come from the burner, converting this kinetic energy into shaft power to drive the compressor and the engine accessories. The turbines can be classified as (1) impulse and (2) reaction. In the impulse turbine, the gases will be expanded in the nozzle and passed over to the moving blades. The moving blades convert this kinetic energy into mechanical energy and also direct the gas flow to the next stage

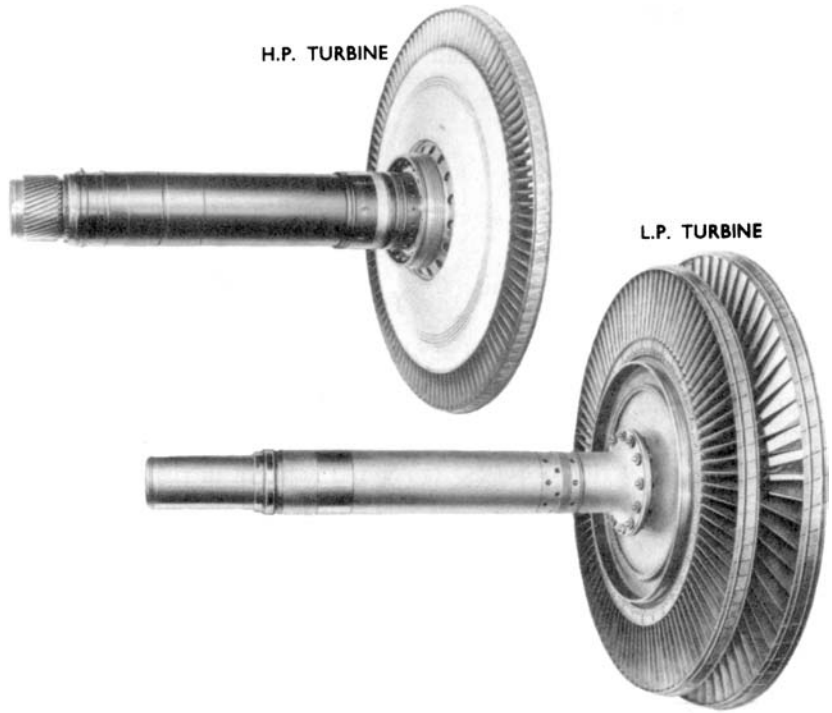

Figure 7.1 Axial flow turbine rotors. (Courtesy Rolls-Royce.)

(multi-stage turbine) or to exit (single-stage turbine). Fig. 7.1 shows the axial flow turbine rotors.

In the case of reaction turbine, pressure drop of expansion takes place in the stator as well as in the rotor-blades. The blade passage area varies continuously to allow for the continued expansion of the gas stream over the rotor-blades. The efficiency of a well-designed turbine is higher than the efficiency of a compressor, and the design process is often much simpler. The main reason for this fact, as discussed in compressor design, is that the fluid undergoes a pressure rise in the compressor. It is much more difficult to arrange for an efficient deceleration of flow than it is to obtain an efficient acceleration. The pressure drop in the turbine is sufficient to keep the boundary layer fluid well behaved, and separation problems, or breakaway of the molecules from the surface, which often can be serious in compressors, can be easily avoided. However, the turbine designer will face much more critical stress problem because the turbine rotors must operate in very high-temperature gases. Since the design principle and concepts of gas turbines are essentially the same as steam turbines, additional information on turbines in general already discussed in Chapter 6 on steam turbines.

7.2 VELOCITY TRIANGLES AND WORK OUTPUT

The velocity diagram at inlet and outlet from the rotor is shown in Fig. 7.2. Gas with an absolute velocity C_1 making an angle α_1, (angle measured from the axial direction) enters the nozzle (in impulse turbine) or stator blades (in reaction turbine). Gas leaves the nozzles or stator blades with an absolute velocity C_2, which makes and an α_2 with axial direction. The rotor-blade inlet angle will be chosen to suit the direction β_2 of the gas velocity V_2 relative to the blade at inlet. β_2 and V_2 are found by subtracting the blade velocity vector U from the absolute velocity C_2.

It is seen that the nozzles accelerate the flow, imparting an increased tangential velocity component. After expansion in the rotor-blade passages, the gas leaves with relative velocity V_3 at angle β_3. The magnitude and direction of the absolute velocity at exit from the rotor C_3 at an angle α_3 are found by vectorial addition of U to the relative velocity V_3. α_3 is known as the swirl angle.

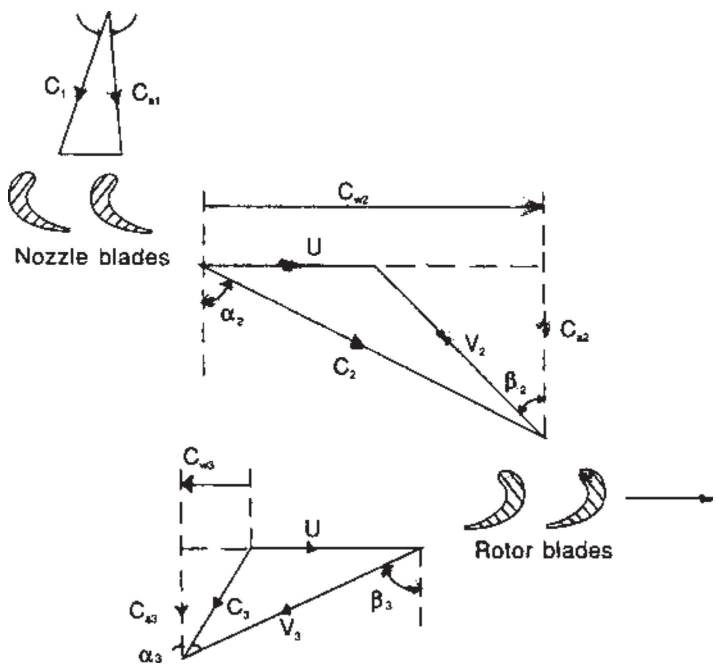

Figure 7.2 Velocity triangles for an axial flow gas turbine.

The gas enters the nozzle with a static pressure p_1 and temperature T_1. After expansion, the gas pressure is p_2 and temperature T_2. The gas leaves the rotor-blade passages at pressure p_3 and temperature T_3. Note that the velocity diagram of the turbine differs from that of the compressor, in that the change in tangential velocity in the rotor, ΔC_w, is in the direction opposite to the blade speed U. The reaction to this change in the tangential momentum of the fluid is a torque on the rotor in the direction of motion. V_3 is either slightly less than V_2 (due to friction) or equal to V_2. But in reaction stage, V_3 will always be greater than V_2 because part of pressure drop will be converted into kinetic energy in the moving blade. The blade speed U increases from root to tip and hence velocity diagrams will be different for root, tip, and other radii points. For short blades, 2-D approach in design is valid but for long blades, 3-D approach in the designing must be considered. We shall assume in this section that we are talking about conditions at the mean diameter of the annulus. Just as with the compressor blading diagram, it is more convenient to construct the velocity diagrams in combined form, as shown in Fig. 7.3. Assuming unit mass flow, work done by the gas is given by

$$W = U(C_{w2} + C_{w3}) \tag{7.1}$$

From velocity triangle

$$\frac{U}{Ca} = \tan\alpha_2 - \tan\beta_2 = \tan\beta_3 - \tan\alpha_3 \tag{7.2}$$

In single-stage turbine, $\alpha_1 = 0$ and $C_1 = Ca_1$. In multi-stage turbine, $\alpha_1 = \alpha_3$ and $C_1 = C_3$ so that the same blade shape can be used. In terms of air angles, the stage

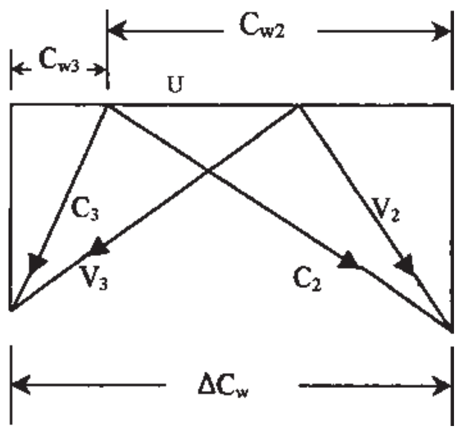

Figure 7.3 Combined velocity diagram.

work output per unit mass flow is given by

$$W = U(C_{w2} + C_{w3}) = UCa(\tan\alpha_2 + \tan\alpha_3) \tag{7.3}$$

or

$$W = UCa\left(\tan\beta_2 + \tan\beta_3\right) \tag{7.4}$$

Work done factor used in the designing of axial flow compressor is not required because in the turbine, flow is accelerating and molecules will not break away from the surface and growth of the boundary layer along the annulus walls is negligible. The stagnation pressure ratio of the stage p_{01}/p_{03} can be found from

$$\Delta T_{0s} = \eta_s T_{01}\left[1 - \left(\frac{1}{p_{01}/p_{03}}\right)^{(\gamma-1)/\gamma}\right] \tag{7.5}$$

where η_s is the isentropic efficiency given by

$$\eta_s = \frac{T_{01} - T_{03}}{T_{01} - T'_{03}} \tag{7.6}$$

The efficiency given by Eq. (7.6) is based on stagnation (or total) temperature, and it is known as total-to-total stage efficiency. Total-to-total stage efficiency term is used when the leaving kinetics energy is utilized either in the next stage of the turbine or in propelling nozzle. If the leaving kinetic energy from the exhaust is wasted, then total-to-static efficiency term is used. Thus total-to-static efficiency,

$$\eta_{ts} = \frac{T_{01} - T_{03}}{T_{01} - T'_3} \tag{7.7}$$

where T'_3 in Eq. (7.7) is the static temperature after an isentropic expansion from p_{01} to p_3.

7.3 DEGREE OF REACTION (Λ)

Degree of reaction is defined as

$$\begin{aligned}\Lambda &= \frac{\text{Enthalpy drop in the moving blades}}{\text{Enthalpy drop in the stage}} \\ &= \frac{h_2 - h_3}{h_1 - h_3} = \frac{Ca}{2U}\left(\tan\beta_1 - \tan\beta_2\right)\end{aligned} \tag{7.8}$$

This shows the fraction of the stage expansion, which occurs in the rotor, and it is usual to define in terms of the static temperature drops, namely

$$\Lambda = \frac{T_2 - T_3}{T_1 - T_3} \tag{7.9}$$

Assuming that the axial velocity is constant throughout the stage, then

$$Ca_2 = Ca_3 = Ca_1, \quad \text{and} \quad C_3 = C_1$$

From Eq. (7.4)

$$C_p(T_1 - T_3) = C_p(T_{01} - T_{03}) = UCa\left(\tan\beta_2 + \tan\beta_3\right) \tag{7.10}$$

Temperature drop across the rotor-blades is equal to the change in relative velocity, that is

$$\begin{aligned} C_p(T_2 - T_3) &= \frac{1}{2}\left(V_3^2 - V_2^2\right) \\ &= \frac{1}{2}Ca^2\left(\sec^2\beta_3 - \sec^2\beta_2\right) \\ &= \frac{1}{2}Ca^2\left(\tan^2\beta_3 - \tan^2\beta_2\right) \end{aligned}$$

Thus

$$\Lambda = \frac{Ca}{2U}\left(\tan\beta_3 - \tan\beta_2\right) \tag{7.11}$$

7.4 BLADE-LOADING COEFFICIENT

The blade-loading coefficient is used to express work capacity of the stage. It is defined as the ratio of the specific work of the stage to the square of the blade velocity—that is, the blade-loading coefficient or temperature-drop coefficient ψ is given by

$$\psi = \frac{W}{\frac{1}{2}U^2} = \frac{2C_p\Delta T_{os}}{U^2} = \frac{2Ca}{U}\left(\tan\beta_2 + \tan\beta_3\right) \tag{7.12}$$

Flow Coefficient (ϕ)

The flow coefficient, ϕ, is defined as the ratio of the inlet velocity Ca to the blade velocity U, i.e.,

$$\phi = \frac{Ca}{U} \tag{7.13}$$

This parameter plays the same part as the blade-speed ratio U/C_1 used in the design of steam turbine. The two parameters, ψ and ϕ, are dimensionless and useful to plot the design charts. The gas angles in terms of ψ, Λ, and ϕ can be obtained easily as given below:

Eqs. (7.11) and (7.12) can be written as

$$\psi = 2\phi(\tan\beta_2 + \tan\beta_3) \tag{7.14}$$

$$\Lambda = \frac{\phi}{2}(\tan\beta_3 - \tan\beta_2) \tag{7.15}$$

Now, we may express gas angles β_2 and β_3 in terms of ψ, Λ, and ϕ as follows:

Adding and subtracting Eqs. (7.14) and (7.15), we get

$$\tan\beta_3 = \frac{1}{2\phi}\left(\frac{1}{2}\psi + 2\Lambda\right) \tag{7.16}$$

$$\tan\beta_2 = \frac{1}{2\phi}\left(\frac{1}{2}\psi - 2\Lambda\right) \tag{7.17}$$

Using Eq. (7.2)

$$\tan\alpha_3 = \tan\beta_3 - \frac{1}{\phi} \tag{7.18}$$

$$\tan\alpha_2 = \tan\beta_2 + \frac{1}{\phi} \tag{7.19}$$

It has been discussed in Chapter 6 that steam turbines are usually impulse or a mixture of impulse and reaction stages but the turbine for a gas-turbine power plant is a reaction type. In the case of steam turbine, pressure ratio can be of the order of 1000:1 but for a gas turbine it is in the region of 10:1. Now it is clear that a very long steam turbine with many reaction stages would be required to reduce the pressure by a ratio of 1000:1. Therefore the reaction stages are used where pressure drop per stage is low and also where the overall pressure ratio of the turbine is low, especially in the case of aircraft engine, which may have only three or four reaction stages.

Let us consider 50% reaction at mean radius. Substituting $\Lambda = 0.5$ in Eq. (7.11), we have

$$\frac{1}{\phi} = \tan\beta_3 - \tan\beta_2 \tag{7.20}$$

Comparing this with Eq. (7.2), $\beta_3 = \alpha_2$ and $\beta_2 = \alpha_3$, and hence the velocity diagram becomes symmetrical. Now considering $C_1 = C_3$, we have $\alpha_1 = \alpha_3 = \beta_2$, and the stator and rotor-blades then have the same inlet and outlet angles. Finally, for $\Lambda = 0.5$, we can prove that

$$\psi = 4\phi\tan\beta_3 - 2 = 4\phi\tan\alpha_2 - 2 \tag{7.21}$$

and

$$\psi = 4\phi\tan\beta_2 + 2 = 4\phi\tan\alpha_3 + 2 \tag{7.22}$$

and hence all the gas angles can be obtained in terms of ψ and ϕ.

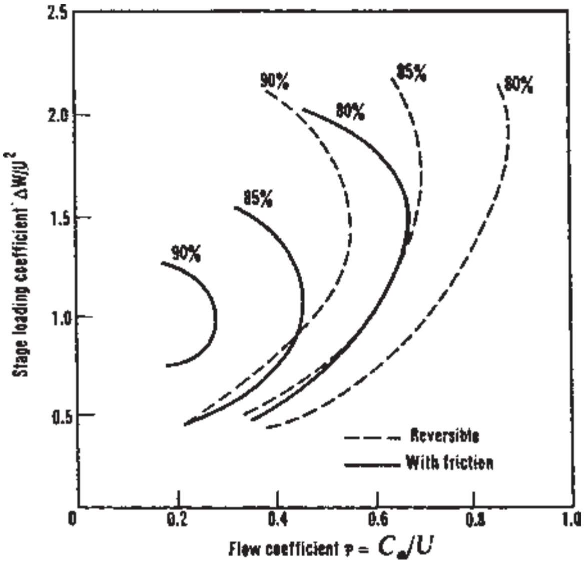

Figure 7.4 Total-to-static efficiency of a 50% reaction axial flow turbine stage.

The low values of ϕ and ψ imply low gas velocities and hence reduced friction losses. But a low value of ψ means more stages for a given overall turbine output, and low ϕ means larger turbine annulus area for a given mass flow. In industrial gas turbine plants, where low sfc is required, a large diameter, relatively long turbine, of low flow coefficient and low blade loading, would be accepted. However, for the gas turbine used in an aircraft engine, the primary consideration is to have minimum weight, and a small frontal area. Therefore it is necessary to use higher values of ψ and ϕ but at the expense of efficiency (see Fig. 7.4).

7.5 STATOR (NOZZLE) AND ROTOR LOSSES

A T–s diagram showing the change of state through a complete turbine stage, including the effects of irreversibility, is given in Fig. 7.5.

In Fig. 7.5, $T_{02} = T_{01}$ because no work is done in the nozzle, $(p_{01} - p_{02})$ represents the pressure drop due to friction in the nozzle. $(T_{01} - T_2^{'})$ represents the ideal expansion in the nozzle, T_2 is the temperature at the nozzle exit due to friction. Temperature, T_2 at the nozzle exit is higher than $T_2^{'}$. The nozzle loss coefficient, λ_N, in terms of temperature may be defined as

$$\lambda_N = \frac{T_2 - T_2^{'}}{C_2^2/2C_p} \qquad (7.23)$$

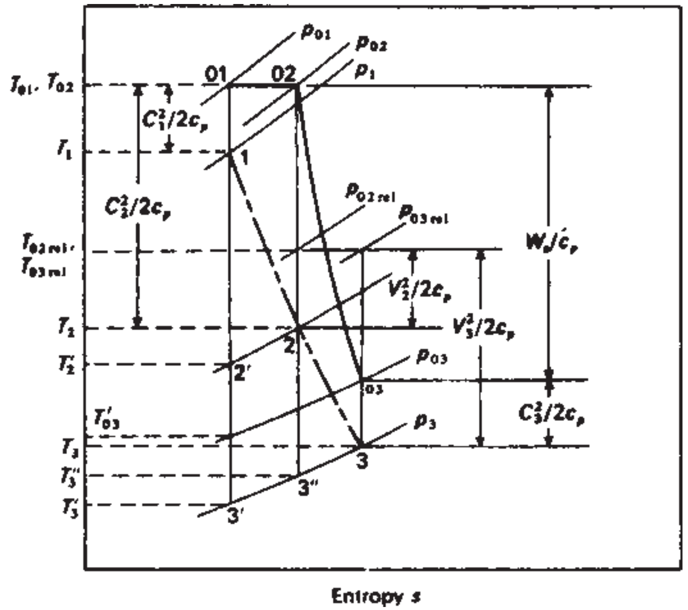

Figure 7.5 T–s diagram for a reaction stage.

Nozzle loss coefficient in term of pressure

$$y_N = \frac{p_{01} - p_{02}}{p_{01} - p_2} \tag{7.24}$$

λ_N and y_N are not very different numerically. From Fig. 7.5, further expansion in the rotor-blade passages reduces the pressure to p_3. $T_3^{'}$ is the final temperature after isentropic expansion in the whole stage, and $T_3^{''}$ is the temperature after expansion in the rotor-blade passages alone. Temperature T_3 represents the temperature due to friction in the rotor-blade passages. The rotor-blade loss can be expressed by

$$\lambda_R = \frac{T_3 - T_3^{''}}{V_3^2/2C_p} \tag{7.25}$$

As we know that no work is done by the gas relative to the blades, that is, $T_{03rel} = T_{02rel}$. The loss coefficient in terms of pressure drop for the rotor-blades is defined by

$$\lambda_R = \frac{p_{02\,rel} - p_{03\,rel}}{p_{03\,rel} - p_3} \tag{7.26}$$

The loss coefficient in the stator and rotor represents the percentage drop of energy due to friction in the blades, which results in a total pressure and static enthalpy drop across the blades. These losses are of the order of 10–15% but can be lower for very low values of flow coefficient.

Nozzle loss coefficients obtained from a large number of turbine tests are typically 0.09 and 0.05 for the rotor and stator rows, respectively. Figure 7.4 shows the effect of blade losses, determined with Soderberg's correlation, on the total-to-total efficiency of turbine stage for the constant reaction of 50%. It is the evident that exit losses become increasingly dominant as the flow coefficient is increased.

7.6 FREE VORTEX DESIGN

As pointed out earlier, velocity triangles vary from root to tip of the blade because the blade speed U is not constant and varies from root to tip. Twisted blading designed to take account of the changing gas angles is called vortex blading. As discussed in axial flow compressor (Chapter 5) the momentum equation is

$$\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{C_{\mathrm{w}}^2}{r} \tag{7.27}$$

For constant enthalpy and entropy, the equation takes the form

$$\frac{\mathrm{d}h_0}{\mathrm{d}r} = Ca\frac{\mathrm{d}Ca}{\mathrm{d}r} + C_{\mathrm{w}}\frac{\mathrm{d}C_{\mathrm{w}}}{\mathrm{d}r} + \frac{C_{\mathrm{w}}^2}{r} \tag{7.28}$$

For constant stagnation enthalpy across the annulus ($\mathrm{d}h_0/\mathrm{d}r = 0$) and constant axial velocity ($\mathrm{d}Ca/\mathrm{d}r = 0$) then the whirl component of velocity C_{w} is inversely proportional to the radius and radial equilibrium is satisfied. That is,

$$C_{\mathrm{w}} \times r = \text{constant} \tag{7.29}$$

The flow, which follows Eq. (7.29), is called a "free vortex."

Now using subscript m to denote condition at mean diameter, the free vortex variation of nozzle angle α_2 may be found as given below:

$$C_{\mathrm{w2}}r = rCa_2 \tan\alpha_2 = \text{constant}$$

$$Ca_2 = \text{constant}$$

Therefore α_2 at any radius r is related to $\alpha_{2\mathrm{m}}$ at the mean radius r_{m} by

$$\tan\alpha_2 = \left(\frac{r_{\mathrm{m}}}{r}\right)_2 \tan\alpha_{2\mathrm{m}} \tag{7.30}$$

Similarly, α_3 at outlet is given by

$$\tan\alpha_3 = \left(\frac{r_{\mathrm{m}}}{r}\right)_3 \tan\alpha_{3\mathrm{m}} \tag{7.31}$$

The gas angles at inlet to the rotor-blade, from velocity triangle,

$$\tan\beta_3 = \tan\alpha_2 - \frac{U}{Ca}$$

$$= \left(\frac{r_m}{r}\right)_2 \tan\alpha_{2m} - \left(\frac{r}{r_m}\right)_2 \frac{U_m}{Ca_2} \quad (7.32)$$

and β_3 is given by

$$\tan\beta_2 = \left(\frac{r_m}{r}\right)_3 \tan\alpha_{3m} + \left(\frac{r}{r_m}\right)_3 \frac{U_m}{Ca_3} \quad (7.33)$$

7.7 CONSTANT NOZZLE ANGLE DESIGN

As before, we assume that the stagnation enthalpy at outlet is constant, that is, $dh_0/dr = 0$. If α_2 is constant, this leads to the axial velocity distribution given by

$$C_{w2} r^{\sin^2\alpha_2} = \text{constant} \quad (7.34)$$

and since α_2 is constant, then Ca_2 is proportional to C_{w1}. Therefore

$$C_{a2} r^{\sin^2\alpha_2} = \text{constant} \quad (7.35)$$

Normally the change in vortex design has only a small effect on the performance of the blade while secondary losses may actually increase.

Illustrative Example 7.1 Consider an impulse gas turbine in which gas enters at pressure = 5.2 bar and leaves at 1.03 bar. The turbine inlet temperature is 1000 K and isentropic efficiency of the turbine is 0.88. If mass flow rate of air is 28 kg/s, nozzle angle at outlet is 57°, and absolute velocity of gas at inlet is 140 m/s, determine the gas velocity at nozzle outlet, whirl component at rotor inlet and turbine work output. Take, $\gamma = 1.33$, and $C_{pg} = 1.147$ kJ/kgK (see Fig. 7.6).

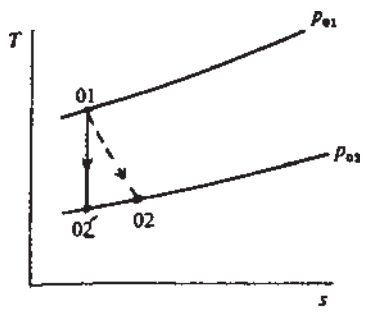

Figure 7.6 *T-s* diagram for Example 7.1.

Solution

From isentropic p–T relation for expansion process

$$\frac{T'_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma}$$

or $$T'_{02} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} = 1000\left(\frac{1.03}{5.2}\right)^{(0.248)} = 669\,\text{K}$$

Using isentropic efficiency of turbine

$$T_{02} = T_{01} - \eta_t\left(T_{01} - T'_{02}\right) = 1000 - 0.88(1000 - 669)$$

$$= 708.72\,\text{K}$$

Using steady-flow energy equation

$$\frac{1}{2}\left(C_2^2 - C_1^2\right) = C_p(T_{01} - T_{02})$$

Therefore, $C_2 = \sqrt{[(2)(1147)(1000 - 708.72) + 19600]} = 829.33\,\text{m/s}$

From velocity triangle, velocity of whirl at rotor inlet

$$C_{w2} = 829.33\ \sin 57° = 695.5\ \ \text{m/s}$$

Turbine work output is given by

$$W_t = mC_{pg}(T_{01} - T_{02}) = (28)(1.147)(1000 - 708.72)$$

$$= 9354.8\ \text{kW}$$

Design Example 7.2 In a single-stage gas turbine, gas enters and leaves in axial direction. The nozzle efflux angle is 68°, the stagnation temperature and stagnation pressure at stage inlet are 800°C and 4 bar, respectively. The exhaust static pressure is 1 bar, total-to-static efficiency is 0.85, and mean blade speed is 480 m/s, determine (1) the work done, (2) the axial velocity which is constant through the stage, (3) the total-to-total efficiency, and (4) the degree of reaction. Assume $\gamma = 1.33$, and $C_{pg} = 1.147$ kJ/kgK.

Solution

(1) The specific work output

$$\begin{aligned} W =\ & C_{pg}(T_{01} - T_{03}) \\ =\ & \eta_{ts} C_{pg} T_{01}\left[1 - (1/4)^{0.33/1.33}\right] \\ =\ & (0.85)(1.147)(1073)\left[1 - (0.25)^{0.248}\right] = 304.42\,\text{kJ/kg} \end{aligned}$$

(2) Since $\alpha_1 = 0$, $\alpha_3 = 0$, $C_{w1} = 0$ and specific work output is given by

$$W = UC_{w2} \quad \text{or} \quad C_{w2} = \frac{W}{U} = \frac{304.42 \times 1000}{480} = 634.21 \text{ m/s}$$

From velocity triangle

$$\sin \alpha_2 = \frac{C_{w2}}{C_2}$$

or

$$C_2 = \frac{C_{w2}}{\sin \alpha_2} = \frac{634.21}{\sin 68°} = 684 \text{ m/s}$$

Axial velocity is given by

$$Ca_2 = 684 \cos 68° = 256.23 \text{ m/s}$$

(3) Total-to-total efficiency, η_{tt}, is

$$\eta_{tt} = \frac{T_{01} - T_{03}}{T_{01} - T'_{03}}$$

$$= \frac{w_s}{T_{01} - \left(T_3 + \frac{C_3^2}{2C_{pg}}\right)} = \frac{w_s}{\frac{w_s}{\eta_{ts}} - \frac{C_3^2}{2C_{pg}}}$$

$$= \frac{304.42}{\frac{304.42}{0.85} - \frac{(256.23)^2}{2 \times 1147}} = 92.4\%$$

(4) The degree of reaction

$$\Lambda = \frac{Ca}{2U}(\tan \beta_3 - \tan \beta_2)$$

$$= \left(\frac{Ca}{2U} \times \frac{U}{Ca}\right) - \left(\frac{Ca}{2U} \tan \alpha_2\right) + \left(\frac{U}{Ca} \times \frac{Ca}{2U}\right)$$

(from velocity triangle)

$$\Lambda = 1 - \frac{Ca}{2U} \tan \alpha_2 = 1 - \frac{256.23}{(2)(480)} \tan 68° = 33.94\%$$

Design Example 7.3 In a single-stage axial flow gas turbine gas enters at stagnation temperature of 1100 K and stagnation pressure of 5 bar. Axial velocity is constant through the stage and equal to 250 m/s. Mean blade speed is 350 m/s. Mass flow rate of gas is 15 kg/s and assume equal inlet and outlet velocities. Nozzle efflux angle is 63°, stage exit swirl angle equal to 9°. Determine the rotor-blade gas angles, degree of reaction, and power output.

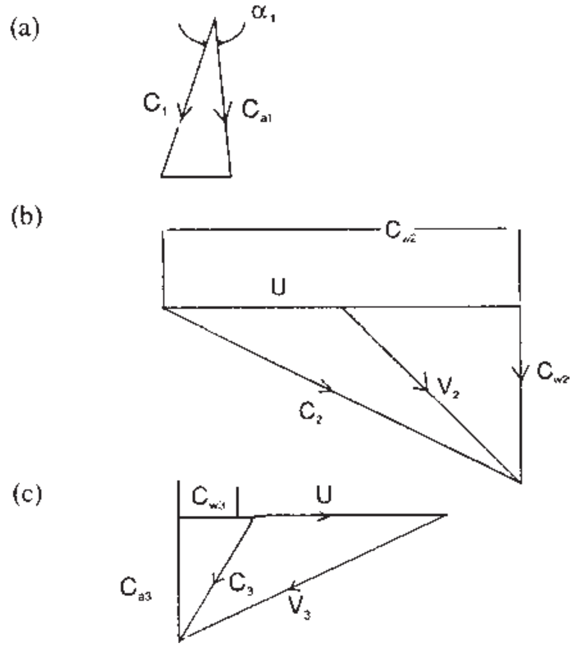

Figure 7.7 Velocity triangles for Example 7.3.

Solution

Refer to Fig. 7.7.

$$Ca_1 = Ca_2 = Ca_3 = Ca = 250 \text{ m/s}$$

From velocity triangle (b)

$$C_2 = \frac{Ca_2}{\cos\alpha_2} = \frac{250}{\cos 63^\circ} = 550.67 \text{ m/s}$$

From figure (c)

$$C_3 = \frac{Ca_3}{\cos\alpha_3} = \frac{250}{\cos 9^\circ} = 253 \text{ m/s}$$

$$C_{w3} = Ca_3 \tan\alpha_3 = 250 \tan 9^\circ = 39.596 \text{ m/s}$$

$$\tan\beta_3 = \frac{U + C_{w3}}{Ca_3} = \frac{350 + 39.596}{250} = 1.5584$$

i.e., $\beta_3 = 57.31^\circ$

From figure (b)

$$C_{w2} = Ca_2 \tan\alpha_2 = 250 \tan 63^\circ = 490.65 \text{ m/s}$$

and

$$\tan\beta_2 = \frac{C_{w2} - U}{Ca_2} = \frac{490.65 - 350}{250} = 0.5626$$

$$\therefore \beta_2 = 29°21'$$

Power output

$$W = mUCa(\tan\beta_2 + \tan\beta_3)$$

$$= (15)(350)(250)(0.5626 + 1.5584)/1000$$

$$= 2784 \text{ kW}$$

The degree of reaction is given by

$$\Lambda = \frac{Ca}{2U}(\tan\beta_3 - \tan\beta_2)$$

$$= \frac{250}{2 \times 350}(1.5584 - 0.5626)$$

$$= 35.56\%$$

Design Example 7.4 Calculate the nozzle throat area for the same data as in the precious question, assuming nozzle loss coefficient, $T_N = 0.05$. Take $\gamma = 1.333$, and $C_{pg} = 1.147$ kJ/kgK.

Solution

Nozzle throat area, $A = m/\rho_2 Ca_2$

and $\rho_2 = \dfrac{p_2}{RT_2}$

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 1100 - \frac{(550.67)^2}{(2)(1.147)(1000)} \quad (T_{01} = T_{02})$$

i.e., $T_2 = 967.81$ K

From nozzle loss coefficient

$$T_2' = T_2 - \lambda_N \frac{C_2^2}{2C_p} = 967.81 - \frac{0.05 \times (550.67)^2}{(2)(1.147)(1000)} = 961.2 \text{ K}$$

Using isentropic $p-T$ relation for nozzle expansion

$$p_2 = p_{01}/\left(T_{01}/T_2'\right)^{\gamma/(\gamma-1)} = 5/(1100/961.2)^4 = 2.915 \text{ bar}$$

Critical pressure ratio

$$p_{01}/p_c = \left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)} = \left(\frac{2.333}{2}\right)^4 = 1.852$$

or $\quad p_{01}/p_2 = 5/2.915 = 1.715$

Since $\frac{p_{01}}{p_2} < \frac{p_{01}}{p_c}$, and therefore nozzle is unchoked.

Hence nozzle gas velocity at nozzle exit

$$C_2 = \sqrt{[2C_{pg}(T_{01} - T_2)]}$$
$$= \sqrt{[(2)(1.147)(1000)(1100 - 967.81)]} = 550.68 \text{ m/s}$$

Therefore, nozzle throat area

$$A = \frac{m}{\rho_2 C_2}, \text{ and } \rho_2 = \frac{p_2}{RT_2} = \frac{(2.915)(10^2)}{(0.287)(967.81)} = 1.05 \text{ kg/m}^3$$

Thus

$$A = \frac{15}{(1.05)(550.68)} = 0.026 \text{ m}^2$$

Design Example 7.5 In a single-stage turbine, gas enters and leaves the turbine axially. Inlet stagnation temperature is 1000 K, and pressure ratio is 1.8 bar. Gas leaving the stage with velocity 270 m/s and blade speed at root is 290 m/s. Stage isentropic efficiency is 0.85 and degree of reaction is zero. Find the nozzle efflux angle and blade inlet angle at the root radius.

Solution

Since $\Lambda = 0$, therefore

$$\Lambda = \frac{T_2 - T_3}{T_1 - T_3},$$

hence

$$T_2 = T_3$$

From isentropic p–T relation for expansion

$$T'_{03} = \frac{T_{01}}{(p_{01}/p_{03})^{(\gamma-1)/\gamma}} = \frac{1000}{(1.8)^{0.249}} = 863.558 \text{ K}$$

Using turbine efficiency

$$T_{03} = T_{01} - \eta_t\left(T_{01} - T'_{03}\right)$$

$$= 1000 - 0.85(1000 - 863.558) = 884\,\text{K}$$

In order to find static temperature at turbine outlet, using static and stagnation temperature relation

$$T_3 = T_{03} - \frac{C_3^2}{2C_{pg}} = 884 - \frac{270^2}{(2)(1.147)(1000)} = 852\,\text{K} = T_2$$

Dynamic temperature

$$\frac{C_2^2}{2C_{pg}} = 1000 - T_2 = 1000 - 852 = 148\,\text{K}$$

$$C_2 = \sqrt{[(2)(1.147)(148)(1000)]} = 582.677 \text{ m/s}$$

Since, $C_{pg}\Delta T_{os} = U(C_{w3} + C_{w2}) = UC_{w2}\ \ (C_{w3} = 0)$

Therefore, $C_{w2} = \dfrac{(1.147)(1000)(1000 - 884)}{290} = 458.8 \text{ m/s}$

From velocity triangle

$$\sin\alpha_2 = \frac{C_{w2}}{C_2} = \frac{458.8}{582.677} = 0.787$$

That is, $\alpha_2 = 51°54'$

$$\tan\beta_2 = \frac{C_{w2} - U}{Ca_2} = \frac{458.8 - 290}{C_2\cos\alpha_2}$$

$$= \frac{458.8 - 290}{582.677\cos 51.90°} = 0.47$$

i.e., $\beta_2 = 25°9'$

Design Example 7.6 In a single-stage axial flow gas turbine, gas enters the turbine at a stagnation temperature and pressure of 1150 K and 8 bar, respectively. Isentropic efficiency of stage is equal to 0.88, mean blade speed is 300 m/s, and rotational speed is 240 rps. The gas leaves the stage with velocity 390 m/s. Assuming inlet and outlet velocities are same and axial, find the blade height at the outlet conditions when the mass flow of gas is 34 kg/s, and temperature drop in the stage is 145 K.

Solution

Annulus area A is given by

$$A = 2\pi r_m h$$

where $h =$ blade height

$r_m =$ mean radius

As we have to find the blade height from the outlet conditions, in this case annulus area is A_3.

$$\therefore \quad h = \frac{A_3}{2\pi r_m}$$

$$U_m = \pi D_m N$$

or $D_m = \frac{(U_m)}{\pi N} = \frac{300}{(\pi)(240)} = 0.398$

i.e., $r_m = 0.199\,\text{m}$

Temperature drop in the stage is given by

$$T_{01} - T_{03} = 145\,\text{K}$$

Hence $T_{03} = 1150 - 145 = 1005\,\text{K}$

$$T_3 = T_{03} - \frac{C_3^2}{2C_{pg}} = 1005 - \frac{390^2}{(2)(1.147)(1000)} = 938.697\ \text{K}$$

Using turbine efficiency to find isentropic temperature drop

$$T'_{03} = 1150 - \frac{145}{0.88} = 985.23\ \text{K}$$

Using isentropic p–T relation for expansion process

$$p_{03} = \frac{p_{01}}{\left(T_{01}/T'_{03}\right)^{\gamma/(\gamma-1)}} = \frac{8}{(1150/985.23)^4} = \frac{8}{1.856}$$

i.e., $p_{03} = 4.31$ bar

Also from isentropic relation

$$p_3 = \frac{p_{03}}{\left(T'_{03}/T_3\right)^{\gamma/(\gamma-1)}} = \frac{4.31}{(985.23/938.697)^4} = \frac{4.31}{1.214} = 3.55\ \text{bar}$$

$$\rho_3 = \frac{p_3}{RT_3} = \frac{(3.55)(100)}{(0.287)(938.697)} = 1.32\ \text{kg/m}^3$$

$$A_3 = \frac{m}{\rho_3 Ca_3} = \frac{34}{(1.32)(390)} = 0.066\ \text{m}^2$$

Finally,

$$h = \frac{A_3}{2\pi r_m} = \frac{0.066}{(2\pi)(0.199)} = 0.053\,\text{m}$$

Design Example 7.7 The following data refer to a single-stage axial flow gas turbine with convergent nozzle:

Inlet stagnation temperature, T_{01}	1100 K
Inlet stagnation pressure, p_{01}	4 bar
Pressure ratio, p_{01}/p_{03}	1.9
Stagnation temperature drop	145 K
Mean blade speed	345 m/s
Mass flow, m	24 kg/s
Rotational speed	14, 500 rpm
Flow coefficient, Φ	0.75
Angle of gas leaving the stage	12°

$$C_{pg} = 1147\,\text{J/kg K}, \quad \gamma = 1.333, \quad \lambda_N = 0.05$$

Assuming the axial velocity remains constant and the gas velocity at inlet and outlet are the same, determine the following quantities at the mean radius:

(1) The blade loading coefficient and degree of reaction
(2) The gas angles
(3) The nozzle throat area

Solution

(1) $$\Psi = \frac{C_{pg}(T_{01} - T_{03})}{U^2} = \frac{(1147)(145)}{345^2} = 1.4$$

Using velocity diagram

$$U/Ca = \tan\beta_3 - \tan\alpha_3$$

or $$\tan\beta_3 = \frac{1}{\Phi} + \tan\alpha_3$$

$$= \frac{1}{0.75} + \tan 12°$$

$$\beta_3 = 57.1°$$

From Equations (7.14) and (7.15), we have

$$\Psi = \Phi\left(\tan\beta_2 + \tan\beta_3\right)$$

and

$$\Lambda = \frac{\Phi}{2}\left(\tan\beta_3 - \tan\beta_2\right)$$

From which

$$\tan\beta_3 = \frac{1}{2\Phi}(\Psi + 2\Lambda)$$

Therefore

$$\tan 57.1^\circ = \frac{1}{2\times 0.75}(1.4 + 2\Lambda)$$

Hence

$$\Lambda = 0.4595$$

(2) $$\tan\beta_2 = \frac{1}{2\Phi}(\Psi - 2\Lambda)$$

$$= \frac{1}{2\times 0.75}(1.4 - [2][0.459])$$

$$\beta_2 = 17.8^\circ$$

$$\tan\alpha_2 = \tan\beta_2 + \frac{1}{\Phi}$$

$$= \tan 17.8^\circ + \frac{1}{0.75} = 0.321 + 1.33 = 1.654$$

$$\alpha_2 = 58.8^\circ$$

(3) $$Ca_1 = U\Phi$$

$$= (345)(0.75) = 258.75 \text{ m/s}$$

$$C_2 = \frac{Ca_1}{\cos\alpha_2} = \frac{258.75}{\cos 58.8^\circ} = 499.49 \text{ m/s}$$

$$T_{02} - T_2 = \frac{C_2^2}{2C_p} = \frac{499.49^2}{(2)(1147)} = 108.76\,\text{K}$$

$$T_2 - T_{2s} = \frac{(T_N)(499.49^2)}{(2)(1147)} = \frac{(0.05)(499.49^2)}{(2)(1147)} = 5.438\,\text{K}$$

$$T_{2s} = T_2 - 5.438$$

$$T_2 = 1100 - 108.76 = 991.24\,\text{K}$$

$$T_{2s} = 991.24 - 5.438 = 985.8\,\text{K}$$

$$\frac{p_{01}}{p_2} = \left(\frac{T_{01}}{T_{2s}}\right)^{\gamma/(\gamma-1)}$$

$$p_2 = 4 \times \left(\frac{985.8}{1100}\right)^4 = 2.58$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{(2.58)(100)}{(0.287)(991.24)} = 0.911\ \mathrm{kg/m^3}$$

(4) Nozzle throat area $= \dfrac{m}{\rho_1 C_1} = \dfrac{24}{(0.907)(499.49)} = 0.053\ \mathrm{m^2}$

$$A_1 = \frac{m}{\rho_1 Ca_1} = \frac{24}{(0.907)(258.75)} = 0.102\ \mathrm{m^2}$$

Design Example 7.8 A single-stage axial flow gas turbine with equal stage inlet and outlet velocities has the following design data based on the mean diameter:

Mass flow	20 kg/s
Inlet temperature, T_{01}	1150K
Inlet pressure	4 bar
Axial flow velocity constant through the stage	255 m/s
Blade speed, U	345 m/s
Nozzle efflux angle, α_2	60°
Gas-stage exit angle	12°

Calculate (1) the rotor-blade gas angles, (2) the degree of reaction, blade-loading coefficient, and power output and (3) the total nozzle throat area if the throat is situated at the nozzle outlet and the nozzle loss coefficient is 0.05.

Solution

(1) From the velocity triangles

$$C_{w2} = Ca \tan \alpha_2$$

$$= 255 \tan 60° = 441.67 \text{ m/s}$$

$$C_{w3} = Ca \tan \alpha_3 = 255 \tan 12° = 55.2 \text{ m/s}$$

$$V_{w2} = C_{w2} - U = 441.67 - 345 = 96.67 \text{ m/s}$$

$$\beta_2 = \tan^{-1} \frac{V_{w2}}{Ca} = \tan^{-1} \frac{96.67}{255} = 20.8°$$

Also $V_{w3} = C_{w3} + U = 345 + 55.2 = 400.2$ m/s

$$\therefore\ \beta_3 = \tan^{-1} \frac{V_{w3}}{Ca} = \tan^{-1} \frac{400.2}{255} = 57.5°$$

(2) $$\Lambda = \frac{\Phi}{2}\left(\tan\beta_3 - \tan\beta_2\right)$$

$$= \frac{255}{2 \times 345}(\tan 57.5^\circ - \tan 20.8^\circ) = 0.44$$

$$\Psi = \frac{Ca}{U}\left(\tan\beta_2 + \tan\beta_3\right)$$

$$= \frac{255}{345}(\tan 20.8^\circ + \tan 57.5^\circ) = 1.44$$

Power $$W = mU(C_{w2} + C_{w3})$$

$$= (20)(345)(441.67 + 54.2) = 3421.5\,\text{kW}$$

(3) $$\lambda_N = \frac{C_p\left(T_2 - T_2'\right)}{\frac{1}{2}C_2^2},\ C_2 = Ca\ \sec\alpha_2 = 255\sec 60^\circ = 510\ \text{m/s}$$

or $$T_2 - T_2' = \frac{(0.05)(0.5)(510^2)}{1147} = 5.67$$

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 1150 - \frac{510^2}{(2)(1147)} = 1036.6\,\text{K}$$

$$T_2' = 1036.6 - 5.67 = 1030.93\,\text{K}$$

$$\frac{p_{01}}{p_2} = \left(\frac{T_{01}}{T_2}\right)^{\gamma/(\gamma-1)} = \left(\frac{1150}{1030.93}\right)^4 = 1.548$$

$$p_2 = \frac{4}{1.548} = 2.584\,\text{bar}$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{2.584 \times 100}{0.287 \times 1036.6} = 0.869\,\text{kg/m}^3$$

$$m = \rho_2 A_2 C_2$$

$$A_2 = \frac{20}{0.869 \times 510} = 0.045\,\text{m}^2$$

Illustrative Example 7.9 A single-stage axial flow gas turbine has the following data

Mean blade speed	340 m/s
Nozzle exit angle	15°
Axial velocity (constant)	105 m/s
Turbine inlet temperature	900°C
Turbine outlet temperature	670°C
Degree of reaction	50%

Calculate the enthalpy drop per stage and number of stages required.

Solution

At 50%,

$$\alpha_2 = \beta_3$$

$$\alpha_3 = \beta_2$$

$$C_2 = \frac{U}{\cos 15°} = \frac{340}{\cos 15°} = 351.99 \text{ m/s}$$

$$\text{Heat drop in blade moving row} = \frac{C_2^2 - C_3^2}{2C_p} = \frac{(351.99)^2 - (105)^2}{(2)(1147)}$$

$$= \frac{123896.96 - 11025}{(2)(1147)}$$

$$= 49.2\,\text{K}$$

$$\text{Therefore heat drop in a stage} = (2)(49.2) = 98.41\,\text{K}$$

$$\text{Number of stages} = \frac{1173 - 943}{98.41} = \frac{230}{98.4} = 2$$

Design Example 7.10 The following particulars relate to a single-stage turbine of free vortex design:

Inlet temperature, T_{01}	1100K
Inlet pressure, p_{01}	4 bar
Mass flow	20 kg/s
Axial velocity at nozzle exit	250 m/s
Blade speed at mean diameter	300 m/s
Nozzle angle at mean diameter	25°
Ratio of tip to root radius	1.4

The gas leaves the stage in an axial direction, find:

(1) The total throat area of the nozzle.
(2) The nozzle efflux angle at root and tip.
(3) The work done on the turbine blades.

Take

$$C_{pg} = 1.147\,\text{kJ/kg K}, \quad \gamma = 1.33$$

Solution

For no loss up to throat

$$\frac{p^*}{p_{01}} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} = \left(\frac{2}{2.33}\right)^4 = 0.543$$

$$p^* = 4 \times 0.543 = 2.172\,\text{bar}$$

Also $T^* = 1100\left(\frac{2}{2.33}\right)^4 = 944\,\text{K}$

$$T_{01} = T^* + \frac{C^2}{2C_{pg}}$$

$$C^* = \sqrt{2C_{pg}\left(T_{01} - T^*\right)}$$

$$= \sqrt{(2)(1147)(1100 - 944)} = 598\ \text{m/s}$$

$$\rho^* = \frac{p^*}{RT^*} = \frac{(2.172)(100)}{(0.287)(944)} = 0.802\,\text{kg/m}^3$$

(1) Throat area

$$A = \frac{m}{\rho C^*} = \frac{20}{(0.802)(598)} = 0.042\,\text{m}^2$$

(2) Angle α_1, at any radius r and $\alpha_{1\text{m}}$ at the design radius r_m are related by the equation

$$\tan \alpha_1 = \frac{r_\text{m}}{r_1} \tan \alpha_{1\text{m}}$$

Given

$$\frac{\text{Tip radius}}{\text{Root radius}} = \frac{r_t}{r_r} = 1.4$$

$$\therefore \quad \frac{\text{Mean radius}}{\text{Root radius}} = 1.2$$

$$\alpha_{1m} = 25^\circ$$

$$\tan\alpha_{1r} = \frac{r_{mean}}{r_{root}} \times \tan\alpha_{1m}$$

$$= 1.2 \times \tan 25^\circ = 0.5596$$

$$\therefore \quad \alpha_{1r} = 29.23^\circ$$

$$\tan\alpha_{1t} = \frac{r_r}{r_t} \times \tan\alpha_{1r} = \left(\frac{1}{1.4}\right)(0.5596) = 0.3997$$

$$\therefore \quad \alpha_{1t} = 21.79^\circ$$

(3) $$C_{w2} = \frac{r_m}{r_r} x C_{w2m} = \frac{r_m}{r_r} \frac{Ca_2}{\tan\alpha_{2m}} = 1.2x \frac{250}{\tan 25^\circ} = 643 \text{ m/s}$$

$$W = mUC_{w2} = \frac{(20)(300)(643)}{1000} = 3858 \text{ kW}$$

7.8 RADIAL FLOW TURBINE

In Sec. 7.1 "Introduction to Axial Flow Turbines", it was pointed out that in axial flow turbines the fluid moves essentially in the axial direction through the rotor. In the radial type the fluid motion is mostly radial. The mixed flow machine is characterized by a combination of axial and radial motion of the fluid relative to the rotor. The choice of turbine depends on the application, though it is not always clear that any one type is superior. For small mass flows, the radial machine can be made more efficient than the axial one. The radial turbine is capable of a high-pressure ratio per stage than the axial one. However, multi-staging is very much easier to arrange with the axial turbine, so that large overall pressure ratios are not difficult to obtain with axial turbines. The radial flow turbines are used in turbochargers for commercial (diesel) engines and fire pumps. They are very compact, the maximum diameter being about 0.2 m, and run at very high speeds. In inward flow radial turbine, gas enters in the radial direction and leaves axially at outlet. The rotor, which is usually manufactured of

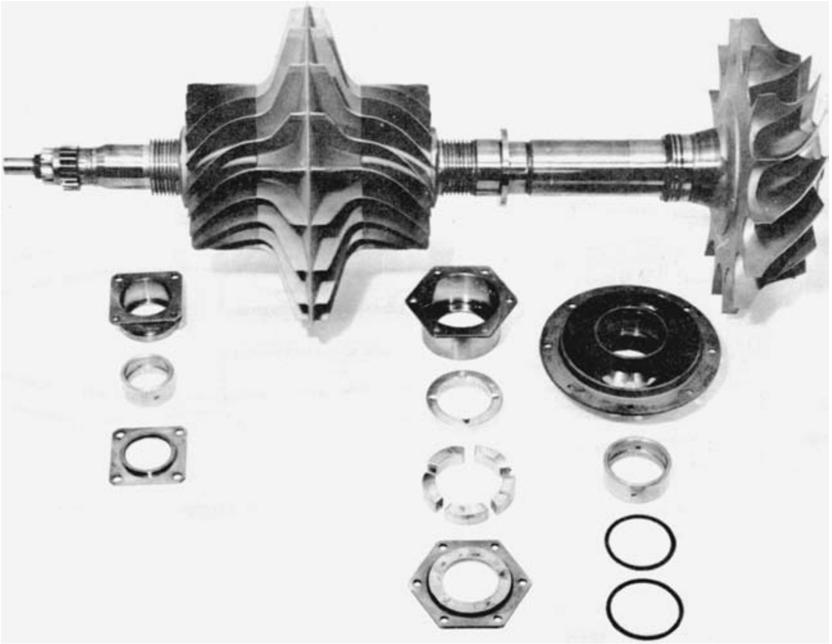

Figure 7.8 Radial turbine photograph of the rotor on the right.

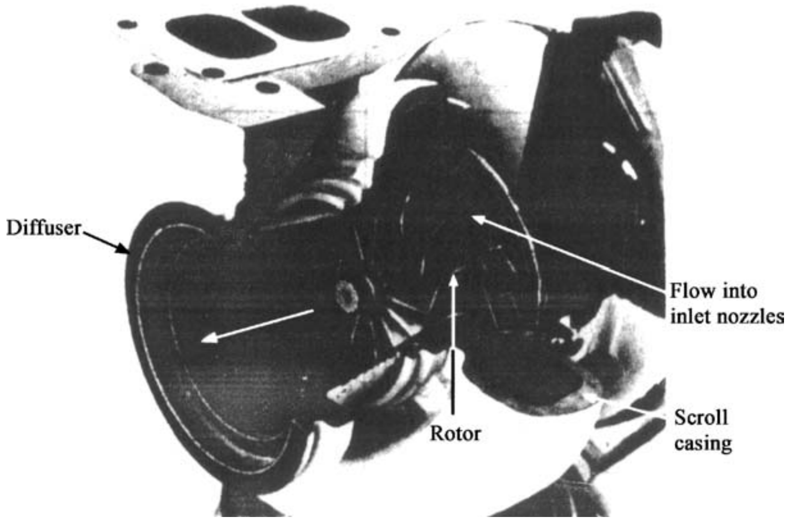

Figure 7.9 Elements of a 90° inward flow radial gas turbine with inlet nozzle ring.

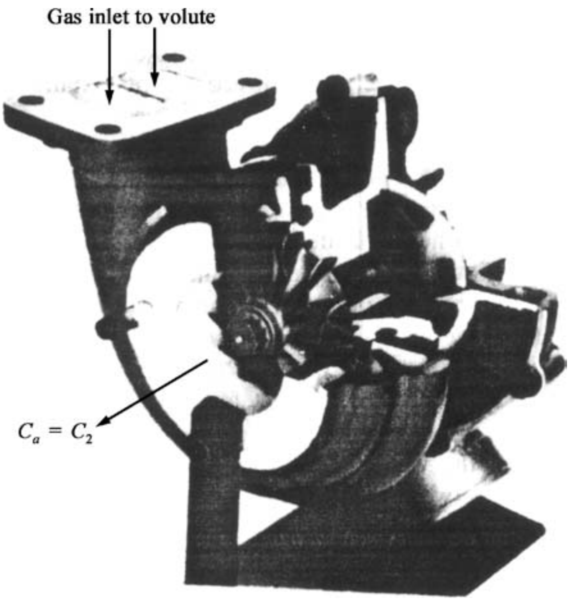

Figure 7.10 A 90° inward flow radial gas turbine without nozzle ring.

cast nickel alloy, has blades that are curved to change the flow from the radial to the axial direction. Note that this turbine is like a single-faced centrifugal compressor with reverse flow. Figures 7.8–7.10 show photographs of the radial turbine and its essential parts.

7.9 VELOCITY DIAGRAMS AND THERMODYNAMIC ANALYSIS

Figure 7.11 shows the velocity triangles for this turbine. The same nomenclature that we used for axial flow turbines, will be used here. Figure 7.12 shows the Mollier diagram for a 90° flow radial turbine and diffuser.

As no work is done in the nozzle, we have $h_{01} = h_{02}$. The stagnation pressure drops from p_{01} to p_1 due to irreversibilities. The work done per unit mass flow is given by Euler's turbine equation

$$W_t = (U_2 C_{w2} - U_3 C_{w3}) \tag{7.36}$$

If the whirl velocity is zero at exit then

$$W_t = U_2 C_{w2} \tag{7.37}$$

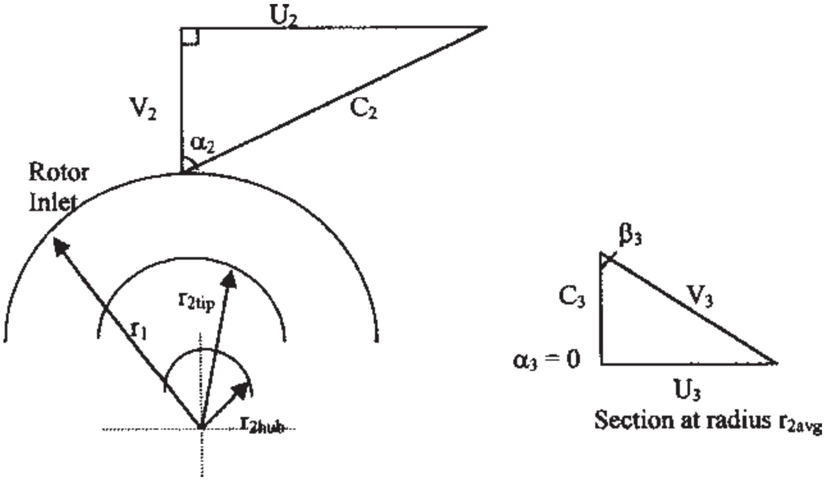

Figure 7.11 Velocity triangles for the 90° inward flow radial gas turbine.

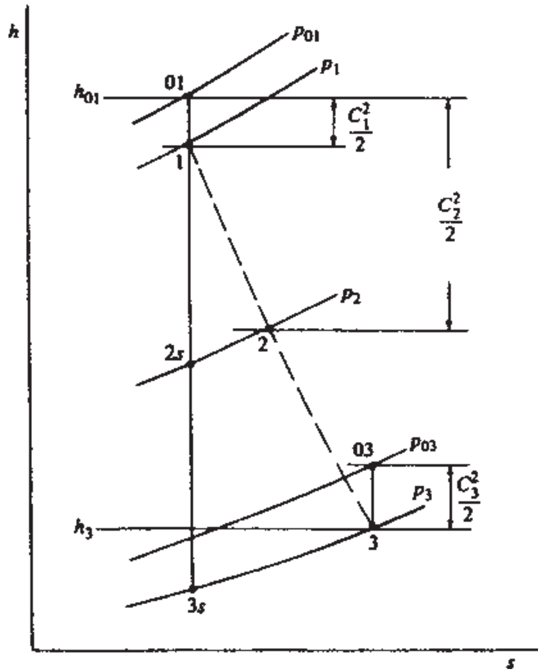

Figure 7.12 Mollier chart for expansion in a 90° inward flow radial gas turbine.

For radial relative velocity at inlet

$$W_t = U_2^2 \tag{7.38}$$

In terms of enthalpy drop

$$h_{02} - h_{03} = U_2 C_{w2} - U_3 C_{w3}$$

Using total-to-total efficiency

$$\eta_{tt} = \frac{T_{01} - T_{03}}{T_{01} - T_{03\,ss}},$$

efficiency being in the region of 80–90%

7.10 SPOUTING VELOCITY

It is that velocity, which has an associated kinetic energy equal to the isentropic enthalpy drop from turbine inlet stagnation pressure p_{01} to the final exhaust pressure. Spouting velocities may be defined depending upon whether total or static conditions are used in the related efficiency definition and upon whether or not a diffuser is included with the turbine. Thus, when no diffuser is used, using subscript 0 for spouting velocity.

$$\frac{1}{2}C_0^2 = h_{01} - h_{03\,ss} \tag{7.39}$$

or

$$\frac{1}{2}C_0^2 = h_{01} - h_{3\,ss} \tag{7.40}$$

for the total and static cases, respectively.

Now for isentropic flow throughout work done per unit mass flow

$$W = U_2^2 = C_0^2/2 \tag{7.41}$$

or

$$U_2/C_0 = 0.707 \tag{7.42}$$

In practice, U_2/C_0 lies in the range $0.68 < \frac{U_2}{C_0} < 0.71$.

7.11 TURBINE EFFICIENCY

Referring to Fig. 7.12, the total-to-static efficiency, without diffuser, is defined as

$$\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3\,ss}}$$

$$= \frac{W}{W + \frac{1}{2}C_3^2 + (h_3 - h_{3ss}) + (h_{3s} - h_{3\,ss})} \tag{7.43}$$

Nozzle loss coefficient, ξ_n, is defined as

$$\xi_n = \frac{\text{Enthalpy loss in nozzle}}{\text{Kinetic energy at nozzle exit}}$$

$$= \frac{h_{3s} - h_{3\,ss}}{0.5C_2^2(T_3/T_2)} \tag{7.44}$$

Rotor loss coefficient, ξ_r, is defined as

$$\xi_r = \frac{h_3 - h_{3s}}{0.5V_3^2} \tag{7.45}$$

But for constant pressure process,

$$T\,ds = dh,$$

and, therefore

$$h_{3s} - h_{3ss} = (h - h_{2s})(T_3/T_2)$$

Substituting in Eq. (7.43)

$$\eta_{ts} = \left[1 + \frac{1}{2}\left(C_3^2 + V_3^2\xi_r + C_2\xi_n T_3/T_2\right)W\right]^{-1} \tag{7.46}$$

Using velocity triangles

$$C_2 = U_2 \operatorname{cosec} \alpha_2, V_3 = U_3 \operatorname{cosec} \beta_3, C_3 = U_3 \cot \beta_3, W = U_2^2$$

Substituting all those values in Eq. (7.44) and noting that $U_3 = U_2\, r_3/r_2$, then

$$\eta_{ts} = \left[1 + \frac{1}{2}\left\{\xi_n \frac{T_3}{T_2}\operatorname{cosec}^2\alpha_2 + \left(\frac{r_3}{r_2}\right)^2\left(\xi_r \operatorname{cosec}^2\beta_3 + \cot^2\beta_3\right)\right\}\right]^{-1} \tag{7.47}$$

Taking mean radius, that is,

$$r_3 = \frac{1}{2}(r_{3t} + r_{3h})$$

Using thermodynamic relation for T_3/T_2, we get

$$\frac{T_3}{T_2} = 1 - \frac{1}{2}\left(\gamma - 1\right)\left(\frac{U_2}{a_2}\right)^2\left[1 - \cot^2\alpha_2 + \left(\frac{r_3}{r_2}\right)^2\cot^2\beta_3\right]$$

But the above value of T_3/T_2 is very small, and therefore usually neglected. Thus

$$\eta_{ts} = \left[1 + \frac{1}{2}\left\{\xi_n \operatorname{cosec}^2\alpha_2 + \left(\frac{r_{3\,av}}{r_2}\right)^2\left(\xi_r \operatorname{cosec}^2\beta_{3\,av} + \cot^2\beta_{3\,av}\right)\right\}\right]^{-1} \tag{7.48}$$

Equation (7.46) is normally used to determine total-to-static efficiency. The η_{ts} can also be found by rewriting Eq. (7.43) as

$$\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3\,ss}} = \frac{(h_{01} - h_{3\,ss}) - (h_{03} - h_3) - (h_3 - h_{3s}) - (h_{3s} - h_{3\,ss})}{(h_{01} - h_{3ss})}$$

$$= 1 - \left(C_3^2 + \xi_n C_2^2 + \xi_r V_3^2\right)/C_0^2 \qquad (7.49)$$

where spouting velocity C_0 is given by

$$h_{01} - h_{3\,ss} = \frac{1}{2}C_0^2 = C_p T_{01}\left[1 - \left(p_3/p_{01}\right)^{\gamma-1/\gamma}\right] \qquad (7.50)$$

The relationship between η_{ts} and η_{tt} can be obtained as follows:

$$W = U_2^2 = \eta_{ts} W_{ts} = \eta_{ts}(h_{01} - h_{3ss}), \text{ then}$$

$$\eta_{tt} = \frac{W}{W_{ts} - \frac{1}{2}C_3^2} = \frac{1}{\frac{1}{\eta_{ts}} - \frac{C_3^2}{2W}}$$

$$\therefore \frac{1}{\eta_{tt}} = \frac{1}{\eta_{ts}} - \frac{C_3^2}{2W} = \frac{1}{\eta_{ts}} - \frac{1}{2}\left(\frac{r_{3\,av}}{r_2} - \cot\beta_{3\,av}\right)^2 \qquad (7.51)$$

Loss coefficients usually lie in the following range for 90° inward flow turbines

$$\xi_n = 0.063\text{–}0.235$$

and

$$\xi_r = 0.384\text{–}0.777$$

7.12 APPLICATION OF SPECIFIC SPEED

We have already discussed the concept of specific speed N_s in Chapter 1 and some applications of it have been made already. The concept of specific speed was applied almost exclusively to incompressible flow machines as an important parameter in the selection of the optimum type and size of unit. The volume flow rate through hydraulic machines remains constant. But in radial flow gas turbine, volume flow rate changes significantly, and this change must be taken into account. According to Balje, one suggested value of volume flow rate is that at the outlet Q_3.

Using nondimensional form of specific speed

$$N_s = \frac{NQ_3^{1/2}}{(\Delta h_0')^{3/4}} \qquad (7.52)$$

where N is in rev/s, Q_3 is in m^3/s and isentropic total-to-total enthalpy drop (from turbine inlet to outlet) is in J/kg. For the 90° inward flow radial turbine,

$U_2 = \pi N D_2$ and $\Delta h_{0s} = \frac{1}{2}C_0^2$, factorizing the Eq. (7.52)

$$N_s = \frac{Q_3^{1/2}}{\left(\frac{1}{2}C_0^2\right)^{3/4}}\left(\frac{U_2}{\pi D_2}\right)\left(\frac{U_2}{\pi N D_2}\right)^{1/2}$$

$$= \left(\frac{\sqrt{2}}{\pi}\right)^{3/2}\left(\frac{U_2}{C_0}\right)^{3/2}\left(\frac{Q_3}{N D_2^3}\right)^{1/2} \tag{7.53}$$

For 90° inward flow radial turbine, $U_2/C_0 = \frac{1}{\sqrt{2}} = 0.707$, substituting this value in Eq. (7.53),

$$N_s = 0.18\left(\frac{Q_3}{N D_2^3}\right)^{1/2}, \quad \text{rev} \tag{7.54}$$

Equation (7.54) shows that specific speed is directly proportional to the square root of the volumetric flow coefficient. Assuming a uniform axial velocity at rotor exit C_3, so that $Q_3 = A_3 C_3$, rotor disc area $A_d = \pi D_2^2/4$, then

$$N = U_2/(\pi D_2) = \frac{C_0\sqrt{2}}{2\pi D_2}$$

$$\frac{Q_3}{N D_2^3} = \frac{A_3 C_3 2\pi D_2}{\sqrt{2} C_0 D_2^2} = \frac{A_3}{A_d}\frac{C_3}{C_0}\frac{\pi^2}{2\sqrt{2}}$$

Therefore,

$$N_s = 0.336\left(\frac{C_3}{C_0}\right)^{\frac{1}{2}}\left(\frac{A_3}{A_d}\right)^{\frac{1}{2}}, \quad \text{rev} \tag{7.55}$$

$$= 2.11\left(\frac{C_3}{C_0}\right)^{\frac{1}{2}}\left(\frac{A_3}{A_d}\right)^{\frac{1}{2}}, \quad \text{rad} \tag{7.56}$$

Suggested values for C_3/C_o and A_3/A_d are as follows:

$$0.04 < C_3/C_0 < 0.3$$

$$0.1 < A_3/A_d < 0.5$$

Then $\quad 0.3 < N_s < 1.1, \quad \text{rad}$

Thus the N_s range is very small and Fig. 7.13 shows the variation of efficiency with N_s, where it is seen to match the axial flow gas turbine over the limited range of N_s.

Design Example 7.11 A small inward radial flow gas turbine operates at its design point with a total-to-total efficiency of 0.90. The stagnation pressure and temperature of the gas at nozzle inlet are 310 kPa and 1145K respectively. The flow leaving the turbine is diffused to a pressure of 100 kPa and the velocity of

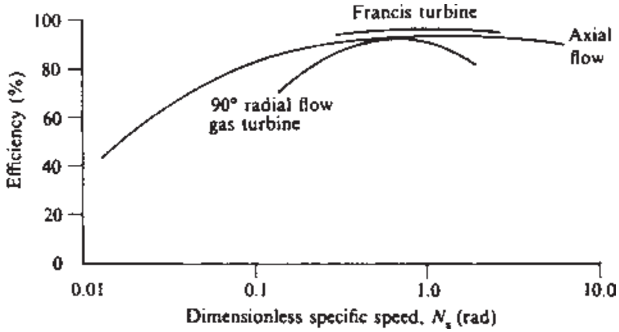

Figure 7.13 Variation of efficiency with dimensionless specific speed.

flow is negligible at that point. Given that the Mach number at exit from the nozzles is 0.9, find the impeller tip speed and the flow angle at the nozzle exit. Assume that the gas enters the impeller radially and there is no whirl at the impeller exit. Take

$$C_p = 1.147 \quad \text{kJ/kg K}, \quad \gamma = 1.333.$$

Solution

The overall efficiency of turbine from nozzle inlet to diffuser outlet is given by

$$\eta_{tt} = \frac{T_{01} - T_{03}}{T_{01} - T_{03\,ss}}$$

Turbine work per unit mass flow

$$W = U_2^2 = C_p(T_{01} - T_{03}), \quad (C_{w3} = 0)$$

Now using isentropic p–T relation

$$T_{01}\left(1 - \frac{T_{03ss}}{T_{01}}\right) = T_{01}\left[1 - \left(\frac{p_{03}}{p_{01}}\right)^{\gamma-1/\gamma}\right]$$

Therefore

$$U_2^2 = \eta_{tt} C_p T_{01}\left[1 - \left(\frac{p_{03}}{p_{01}}\right)^{\gamma-1/\gamma}\right]$$

$$= 0.9 \times 1147 \times 1145\left[1 - \left(\frac{100}{310}\right)^{0.2498}\right]$$

$\therefore$ Impeller tip speed, $U_2 = 539.45$ m/s

The Mach number of the absolute flow velocity at nozzle exit is given by

$$M = \frac{C_1}{a_1} = \frac{U_1}{a_1 \sin \alpha_1}$$

Since the flow is adiabatic across the nozzle, we have

$$T_{01} = T_{02} = T_2 + \frac{C_2^2}{2C_p} = T_2 + \frac{U_2^2}{2C_p \sin^2 \alpha_2}$$

or $$\frac{T_2}{T_{01}} = 1 - \frac{U_2^2}{2C_p T_{01} \sin^2 \alpha_2}, \text{ but } C_p = \frac{\gamma R}{\gamma - 1}$$

$$\therefore \frac{T_2}{T_{01}} = 1 - \frac{U_2^2(\gamma - 1)}{2\gamma R T_{01} \sin^2 \alpha_2} = 1 - \frac{U_2^2(\gamma - 1)}{2a_{01}^2 \sin^2 \alpha_2}$$

But $$\left(\frac{T_2}{T_{01}}\right)^2 = \frac{a_2}{a_{01}} = \frac{a_2}{a_{02}} \qquad \text{since } T_{01} = T_{02}$$

and $$\frac{a_2}{a_{02}} = \frac{U_2}{M_2 a_{02} \sin \alpha_2}$$

$$\therefore \left(\frac{U_2}{M_2 a_{02} \sin \alpha_2}\right)^2 = 1 - \frac{U_2^2(\gamma - 1)}{2a_{02}^2 \sin^2 \alpha_2}$$

and $$1 = \left(\frac{U_2}{a_{02} \sin \alpha_2}\right)^2 \left(\frac{(\gamma - 1)}{2} + \frac{1}{M_2^2}\right)$$

or $$\sin^2 \alpha_2 = \left(\frac{U_2}{a_{02}}\right)^2 \left(\frac{(\gamma - 1)}{2} + \frac{1}{M_2^2}\right)$$

But $$a_{02}^2 = \gamma R T_{02} = (1.333)(287)(1145) = 438043 \text{ m}^2/\text{s}^2$$

$$\therefore \sin^2 \alpha_2 = \frac{539.45^2}{438043} \left(\frac{0.333}{2} + \frac{1}{0.9^2}\right) = 0.9311$$

Therefore nozzle angle $\alpha_2 = 75°$

Illustrative Example 7.12 The following particulars relate to a small inward flow radial gas turbine.

Rotor inlet tip diameter	92 mm
Rotor outlet tip diameter	64 mm
Rotor outlet hub diameter	26 mm
Ratio C_3/C_0	0.447

Ratio U_2/C_0 (ideal)	0.707
Blade rotational speed	30, 500 rpm
Density at impeller exit	$1.75\,\text{kg/m}^3$

Determine

(1) The dimensionless specific speed of the turbine.
(2) The volume flow rate at impeller outlet.
(3) The power developed by the turbine.

Solution

(1) Dimensionless specific speed is

$$N_s = 0.336\left(\frac{C_3}{C_0}\right)^{\frac{1}{2}}\left(\frac{A_3}{A_d}\right)^{\frac{1}{2}},\ \text{rev}$$

Now

$$A_3 = \frac{\pi\left(D_{3t}^2 - D_{3h}^2\right)}{4}$$
$$= \frac{\pi\left(0.064^2 - 0.026^2\right)}{4} = (2.73)(10^{-3})\ \text{m}^2$$
$$A_d = \frac{\pi D_2^2}{4} = \left(\frac{\pi}{4}\right)(0.092^2) = (6.65)(10^{-3})\ \text{m}^2$$

Dimensionless specific speed

$$N_s = 0.336\left(\frac{[0.447][2.73]}{6.65}\right)^{\frac{1}{2}}$$
$$= 0.144\ \text{rev}$$
$$= 0.904\ \text{rad}$$

(2) The flow rate at outlet for the ideal turbine is given by Eq. (7.54).

$$N_s = 0.18\left(\frac{Q_3}{ND_2^3}\right)^{1/2}$$
$$0.144 = 0.18\left(\frac{[Q_3][60]}{[30,500][0.092^3]}\right)^{1/2}$$

Hence

$$Q_3 = 0.253\,\text{m}^3/\text{s}$$

(3) The power developed by the turbine is given by

$$
\begin{aligned}
W_t &= \dot{m}U_3^2 \\
&= \rho_3 Q_3 U_3^2 \\
&= 1.75 \times 0.253 \times \left(\frac{\pi N D_2}{60}\right)^2 \\
&= 1.75 \times 0.253 \times \left(\frac{[\pi][30,500][0.092]}{60}\right)^2 \\
&= 9.565 \text{ kW}
\end{aligned}
$$

PROBLEMS

7.1 A single-stage axial flow gas turbine has the following data:

Inlet stagnation temperature	1100K
The ratio of static pressure at the nozzle exit to the stagnation pressure at the nozzle inlet	0.53
Nozzle efficiency	0.93
Nozzle angle	20°
Mean blade velocity	454 m/s
Rotor efficiency	0.90
Degree of reaction	50%

$$C_{pg} = 1.147 \text{ kJ/kgK}, \quad \gamma = 1.33$$

Find (1) the work output per kg/s of air flow, (2) the ratio of the static pressure at the rotor exit to the stagnation pressure at the nozzle inlet, and (3) the total-to-total stage efficiency.

(282 kW, 0.214, 83.78%)

7.2 Derive an equation for the degree of reaction for a single-stage axial flow turbine and show that for 50% reaction blading $\alpha_2 = \beta_3$ and $\alpha_3 = \beta_2$.

7.3 For a free-vortex turbine blade with an impulse hub show that degree of reaction

$$\Lambda = 1 - \left(\frac{r_h}{r}\right)^2$$

where r_h is the hub radius and r is any radius.

7.4 A 50% reaction axial flow gas turbine has a total enthalpy drop of 288 kJ/kg. The nozzle exit angle is 70°. The inlet angle to the rotating blade row is inclined at 20° with the axial direction. The axial velocity is constant through the stage. Calculate the enthalpy drop per row of moving blades and the number of stages required when mean blade speed is 310 m/s. Take $C_{pg} = 1.147$ kJ/kgK, $\gamma = 1.33$.

(5 stages)

7.5 Show that for zero degree of reaction, blade-loading coefficient, $\Psi = 2$.

7.6 The inlet stagnation temperature and pressure for an axial flow gas turbine are 1000K and 8 bar, respectively. The exhaust gas pressure is 1.2 bar and isentropic efficiency of turbine is 85%. Assume gas is air, find the exhaust stagnation temperature and entropy change of the gas.

(644K, −0.044 kJ/kgK)

7.7 The performance date from inward radial flow exhaust gas turbine are as follows:

Stagnation pressure at inlet to nozzles, p_{01}	705 kPa
Stagnation temperature at inlet to nozzles, T_{01}	1080K
Static pressure at exit from nozzles, p_2	515 kPa
Static temperature at exit from nozzles, T_2	1000K
Static pressure at exit from rotor, p_3	360 kPa
Static temperature at exit from rotor, T_3	923K
Stagnation temperature at exit from rotor, T_{03}	925K
Ratio $\frac{r_{2\,av}}{r_2}$	0.5
Rotational speed, N	25,500 rpm

The flow into the rotor is radial and at exit the flow is axial at all radii. Calculate (1) the total-to-static efficiency of the turbine, (2) the impeller tip diameter, (3) the enthalpy loss coefficient for the nozzle and rotor rows, (4) the blade outlet angle at the mean diameter, and (5) the total-to-total efficiency of the turbine.

[(1) 93%, (2) 0.32 m, (3) 0.019, 0.399, (4) 72.2°, (5) 94%]

NOTATION

A	area
C	absolute velocity

C_0	spouting velocity
h	enthalpy, blade height
N	rotation speed
N_s	specific speed
P	pressure
r_m	mean radius
T	temperature
U	rotor speed
V	relative velocity
Y_N	nozzle loss coefficient in terms of pressure
α	angle with absolute velocity
β	angle with relative velocity
ΔT_{0s}	stagnation temperature drop in the stage
ΔT_s	static temperature drop in the stage
ε_n	nozzle loss coefficient in radial flow turbine
ε_r	rotor loss coefficient in radial flow turbine
ϕ	flow coefficient
η_s	isentropic efficiency of stage
Λ	degree of reaction